ON THE CONSIDERATION OF INITIAL PHASE AND INTENSITY PROFILES IN X-RAY PHASE CONTRAST VISUALIZATION

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The work is devoted to analytical calculations of the evolution of an intense wave field during 2-D visualization in an X-ray phase contrast experiment to study the internal structure of a homogeneous body. The well-known non-iterative algorithm for solving the inverse phase recovery problem was modified by generalizing the initial conditions for the differential transport of intensity equation. An analytical expression for estimating the projected thickness of the studied sample was obtained using the integral Fourier representation of intensities.

Imaging of objects, especially hidden from the naked eye, plays an indispensable role in transport safety systems, health diagnostics (the degree of progression of a particular disease), predicting the lifetime (strength) of devices and quality control of various high-speed dynamical technological processes (laser welding, needle-free injections, etc.). Sometimes it is necessary to use alternative visualization methods to the existing ones due to contraindications or as an additional source of information. In the case of X-ray examination, the absorbed dose and the contrast of the obtained images are crucial, especially in the case of medical imaging.

Phase contrast research methods have been indispensable sources of information for the past decades, allowing for more accurate classification or differentiation of substances, materials, tissues that are similar in atomic composition. Such experimental techniques require X-ray sources with much better brightness and radiation-emitting area of the device. Refractive, reflective, diffractive optical elements are components of any research equipment in addition to a specialized digital detector that records the intensity data and transmits them for further analysis and post-processing.

In the considered propagation-based X-ray phase contrast method, as in other experimental techniques of this class, except for interferometric methods, a wave phase signal is generated during its propagation and interaction of coherent radiation, the intensity of which is recorded by a detector at a certain distance R_2 from the source, with the substance of an object. The intensity recorded in the experiment is the input data to this inverse problem [3] of phase retrieval.

The present work aims to investigate the available computer approaches to solving the inverse problem in X-ray phase contrast imaging and to generalize the known algorithm [1] in case when, additionally, the profiles of the intensity I_0 and phase φ_0 distributions of the probing radiation in the plane Oxy perpendicular to the optical axis Oz, immediately behind the object of study are known. The object of study in such a 2-dimensional imaging is characterized by its projected thickness in the plane where a high-energy detector is placed.

The wave field propagating in space interacts with the substance of the investigated body/sample, which is generally described by the complex refractive index $n(x, y, z) = 1 - \delta + i\beta$ must obey the Helmholtz equation [3]. We solved the differential transport of intensity equation(TIE) [1], which describes the local concentration of the optical flux that is propagating along the optical axis Oz:

$$\nabla_\perp[I(x, y, z)\nabla_\perp\varphi(x, y, z)] = -k\frac{\partial I}{\partial z}, \tag{1}$$

where $\nabla_\perp$ is the gradient in the plane (x, y) and the constant $k = \frac{2\pi}{\lambda}$is the wave number, which is related to the X-ray wavelength λ. Firstly, modified the initial conditions for the phase and for the intensity right after the theoretically investigated complex object, e.g. at the plane where $z = 0$:

$$\varphi(x, y, 0) = \varphi_0(x, y) - k\delta \cdot T(x, y), \tag{2}$$

$$I_0(x, y, 0) = I_0(x, y)e^{-\mu T(x,y)}, \tag{3}$$

in which δ and β are the optical characteristic responsible for the refractive and absorptive properties of a homogeneous sample, and μ is the linear absorption coefficient. By generalizing the initial intensity $I_0 = I_0(x, y)$ and taking into account the initial phase $\varphi_0(x, y)$ of the wave field, we can find the solution of the initial differential TIE (eq.1) for the projected thickness of the object $T(x, y)$.

Now, act by the gradient operator $\nabla_\perp$ on the product $I(x, y, z)\nabla_\perp\varphi(x, y, z)$ and apply the finite-difference representation of the partial derivative $\frac{\partial I}{\partial z} \simeq \frac{[I(x,y,z+\Delta z)]}{\Delta z}$ for the right-hand side of the equation (1). Using the method of Fourier integral representation of both the initial and the one downstream the optical axis (registered by the detector) intensities, as proposed in the reference work, we obtained the operator form equation that is linear for the projected thickness $T(x, y)$:

$$e^{-\mu T(x,y)} = \frac{I(R_2)}{\nabla_\perp\big(I_0(x,y)\big)\Big[\frac{-R_2\cdot\delta}{\mu}\nabla_\perp - \frac{k}{R_2}\varphi_0(x,y)\Big] + I_0(x,y)\Big[1 - \frac{R_2\cdot\delta}{\mu}\nabla^2 - \frac{k}{R_2}\nabla_\perp\varphi_0(x,y) - \frac{k}{R_2}\varphi_0(x,y)\Big]}. \tag{4}$$

Accordingly, by taking the logarithm of both parts of the above expression and by multiplying them on the value $-\frac{1}{\mu}$, we obtain the desired relation:

$$T(x,y) = -\frac{1}{\mu} ln\left(\mathcal{F}^{-1}\left(\frac{\mathcal{F}(I(R_2))}{\mathcal{F}\left\{\nabla_\perp\left(I_0(x,y)\right)\left[\frac{-R_2\cdot\delta}{\mu}\nabla_\perp - \frac{k}{R_2}\varphi_0(x,y)\right] + I_0(x,y)\left[1 - \frac{R_2\cdot\delta}{\mu}\nabla_\perp^2 - \frac{k}{R_2}\nabla_\perp\varphi_0(x,y) - \frac{k}{R_2}\varphi_0(x,y)\right]\right\}} \right) \right). \tag{5}$$

Thus, the algorithm designed for quantitative phase recovery based on only one normalized image was modified. The last equation is our final result for finding the projected thickness of the studied homogeneous sample with the generalized profiles of the wavefront phase (eq.2) and the intensity (eq.3), acquired through the utilization of the Fourier and inverse Fourier transform $\mathcal{F}$ and $\mathcal{F}^{-1}$ respectively.

For the correct operation and exploring of our modified algorithm, represented here in the last equation, it is necessary to establish the features of the action of the gradient operator in the denominator (eq.5) on the corresponding characteristics of the wave field using Fourier derivative theorem. For this reason it is necessary to understand all of the possible forms for the general intensity profile $I_0(x,y)$ and the phase delay $\varphi_0(x,y)$.

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