FIRST-ORDER QED PROCESSES FOR VIRTUAL PHOTONIC STATES IN THE FIELD OF A LINEAR-POLARIZED WAVE

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In order to construct a theory of 2nd-order processes through probabilities describing 1st-order processes, a theory of 1st-order processes with virtual photon states was developed

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INTRODUCTION

Fig. 1 shows all possible 1st order processes that can occur in the field of an electromagnetic wave:

a) Photon emission by an electron during scattering in an electromagnetic wave field.
b) Annihilation of an electron-positron pair in the field of a monochromatic electromagnetic wave.
c) Formation of an electron-positron pair by a photon in the field of a monochromatic electromagnetic wave.
d) Absorption of a photon by an electron in the field of a monochromatic electromagnetic wave.

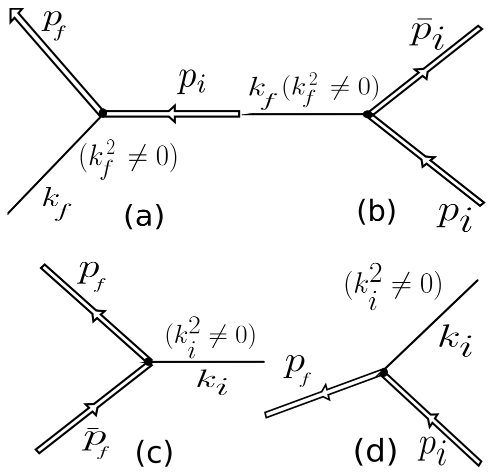

Fig. 1. Feynman diagrams of 1st order processes in a wave field

Fig. 2 shows diagrams of 2nd-order processes with a fine structure constant with a photonic intermediate state, which are described by one diagram. Although it may seem like these diagrams can be composed of the diagrams of the 1st order process from Figure 1 and this is at the amplitude level, during the calculation of the probability in standard way [1] due to the procedure of finding the spur, the terms are mixed and the possibility of writing the probability of the 2nd-order process through to the 1st-order probabilities disappears. However, there is a possibility to express the probabilities of 2nd order processes in terms of 1st order probabilities, but these probabilities must be written in terms of first order probabilities that take into account the polarization properties of intermediate states and are obtained for intermediate states are out the mass surface. Due to these considerations, it is a separate task to obtain these probabilities.

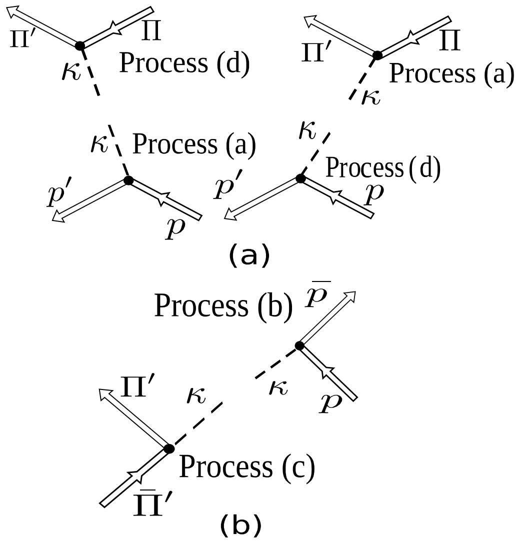

Fig. 2. Feyman diagrams of processes of the 2nd order according to the constant of the fine structure with a photon intermediate state described by one diagram: a – the process of electron scattering by a muon; b – the process of converting an e^-e^+ pair into a muon pair

AMPLITUDE

The amplitude of a process of photon emission by an electron in a wave field (see Fig. 1a) after integration over the 4-coordinate of the vertex can be written as:

$$S_{fi} = C\sum_{l=-\infty}^{\infty}\left(e^*_{f,\mu}A^{\mu}\right)\delta^{(4)}(\tilde{p}_i + lk - \tilde{p}_f - k_f), \quad (1)$$

$$\left(e^*_{f,\mu}A^{\mu}\right) = \bar{u}_{p_f}\left(e^*_{f,\nu}M_l^{\nu}(p_f, p_i)\right)u_{p_i}, \quad (2)$$

$$A_{\mu} \equiv A_{\mu}(p_f, p_i) = \bar{u}_{p_f}M_{l,\mu}(p_f, p_i)u_{p_i}, \quad (3)$$

де позначено

$$\left(e^*_{f,\nu}M_l^{\nu}(2)\right) = \hat{b} - i\beta\gamma^5\hat{k}, \quad (4)$$

$$b = L_l e^*_f + \frac{m}{4}\beta_+ D_+\left(e_x e^*_f\right)k,\ \beta = \frac{m}{4}\beta_- D_+\left(e_y e^*_f\right), \quad (5)$$

$$\beta_{\pm} = \frac{1}{\left(kp_f\right)} \pm \frac{1}{\left(kp_i\right)}, \quad (6)$$

$$L_l = \sum_{s=-\infty}^{\infty} J_{l-2s}(y)J_s(z), \quad D_+ = \eta\left(L_{l+1} + L_{l-1}\right), \quad (7)$$

$$y = 2l\eta\frac{m}{\tilde{m}}\sqrt{\rho\left(1-\rho-\beta_{res}\right)}\cos\psi,\ \rho = \frac{\tilde{m}^2\beta_-}{2l}, \quad (8)$$

$$g=\frac{p_f}{(kp_f)}-\frac{p_i}{(kp_i)}, \qquad (9)$$

$$z=l\left(\eta\frac{m}{\tilde{m}}\right)^2\frac{\rho}{4},\ \beta_{res}=\frac{k_f^2}{4\upsilon_l'\tilde{m}^2},\ \upsilon_l'=\frac{l(kk_f)}{2\tilde{m}^2}, \qquad (10)$$

where k_f, e_f are 4-momentum and 4-vector of polarisation of the emitted photon, p_i, p_f are 4-momentums of the initial and final electrons; the tilde above the 4-momentum indicates the corresponding quasimomentum [1, 2], u_p is the bispinor of a free electron; C is the normalizing constant, a parameter β_{res} determines the deviation of the photon state from the mass surface.

Contrary to [2] in expressions (1)-(10) we consider that $k_f^2\neq 0$.

Note that the amplitudes of other 1st order processes in terms of the fine structure constant can be obtained from (1)-(10) due to the cross-invariance of these processes by simple substitutions.

PROBABILITY

The differential probability of the process of a photon emission by an electron in a wave field per unit time can be written in the form of partial components:

$$d\mathrm{w}_{fi}=\sum_{l=1}^{\infty}dw_{fi}^{(l)}. \qquad (11)$$

The partial probability is given by

$$d\mathrm{w}_{fi}^{(l)}=\left|S_{fi}^{(l)}\right|^2\frac{V^2d^3k_fd^3p_f}{T(2\pi)^6}=$$

$$=\mathrm{w}_0\mathrm{W}\delta^{(4)}\left(\tilde{p}_i+lk-\tilde{p}_f-k_f\right)d^3k_fd^3p_f, \qquad (12)$$

$$\mathrm{W}=\left|\left(e_f^*A\right)\right|^2, \qquad (13)$$

where V is the normalizing volume, T is the infinite time interval, w_0 is the normalizing constant

To describe the polarization properties of a virtual photon state, consider an orthonormal basis in 4-dimensional space:

$$e_0'=n_f,e_1',e_2',e_3'=\lambda\frac{k}{(kn_f)}+n_f, \qquad (14)$$

$$n_f=\frac{k_f}{\sqrt{\left|k_f^2\right|}},\ e_1'=e_x-\frac{(k_fe_x)}{(kk_f)}k, \qquad (15)$$

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$$e_2'=e_y-\frac{(k_fe_y)}{(kk_f)}k,\ \lambda=e_0'^2=\mp 1, \qquad (16)$$

where $k=(\omega,\mathbf{k})=\omega(e_0+e_3)$ is the wave vector.

The 4-vector (3) expanded in basis (14) has three components:

$$A=-(Ae_1')e_1'-(Ae_2')e_2'-\lambda(Ae_3')e_3', \qquad (17)$$

which corresponds to the three possible polarizations of the virtual photon. The vectors e_1', e_2' coincide with the polarization vectors of a real photon, and e_3' corresponds to the additional polarization of the virtual photon. The vector e_0' distribution in this formula is absent due to the orthogonality of the photon and electron currents:

$$(k_fA)\equiv(n_fA)=0. \qquad (18)$$

Taking into account (17), we obtain:

$$\left|(e_fA)\right|^2=-\frac{|A|^2}{3}+\sum_{i=1}^{2}\sum_{j=i+1}^{3}\left(\frac{\xi_1^{(ij)}}{3}\left(\left|(e_i'A)\right|^2-\left|(e_j'A)\right|^2\right)+\right.$$

$$+\frac{1}{2}\left(\xi_2^{(ij)}\left(\left|\left(e_{+\pi/4}^{(ij)}A\right)\right|^2-\left|\left(e_{-\pi/4}^{(ij)}A\right)\right|^2\right)+\right.$$

$$\left.\left.+\xi_3^{(ij)}\left(\left|\left(e_-^{(ij)}A\right)\right|^2-\left|\left(e_+^{(ij)}A\right)\right|^2\right)\right)\right), \qquad (19)$$

where a generalization of Stokes parameters for the case of 3 polarizations has been introduced:

$$\xi_1^{(ij)}=\left|\left(e_f^*e_i'\right)\right|^2-\left|\left(e_f^*e_j'\right)\right|^2,\ \xi_2^{(ij)}=2\Re\left(\left(e_f^*e_i'\right)\left(e_fe_j'\right)\right),$$

$$\xi_3^{(ij)}=2\Im\left(\left(e_f^*e_i'\right)\left(e_fe_j'\right)\right), \qquad (20)$$

where $e_{\pm\pi/4}^{(ij)}$ are the polarization, corresponding to polarization at an angle $\pm 45^{\circ}$, $e_{\pm}^{(ij)}$ corresponds to the circular polarizations of a virtual photon in the plane $e_i'e_j'$ (there are only three such planes: $e_1'e_2'$, $e_1'e_3'$, $e_2'e_3'$):

$$e_{\pm\pi/4}^{(ij)}=\frac{1}{\sqrt{2}}\left(e_i'\pm e_j'\right),\quad e_{\pm}^{(ij)}=\frac{1}{\sqrt{2}}\left(e_i'\pm ie_j'\right), \qquad (21)$$

$$\mathrm{W}=\frac{1}{3}\mathrm{w}_{Unpolar}+\sum_{i=1}^{2}\sum_{j=i+1}^{3}\left(\frac{1}{3}\xi_1^{(ij)}\left(\mathrm{w}(e_i')-\mathrm{w}(e_i')\right)+\right.$$

$$+\frac{1}{2}\xi_2^{(ij)}\left(\mathrm{w}\left(e_{+\pi/4}^{(ij)}\right)-\mathrm{w}\left(e_{-\pi/4}^{(ij)}\right)\right)+$$

$$\left.+\frac{1}{2}\xi_3^{(ij)}\left(\mathrm{w}\left(e_+^{(ij)}\right)-\mathrm{w}\left(e_-^{(ij)}\right)\right)\right), \qquad (22)$$

where

$$\mathrm{w}(e_j)=\frac{1}{4}\left\{\mathrm{Sp}\left(\hat{p}_f+m\right)\left(e^*M_l(p_f,p_i)\right)\times\right.$$

$$\left.\times\left(\hat{p}_i+m\right)\overline{\left(e^*M_l(p_f,p_i)\right)}\right\}. \qquad (23)$$

Calculation (23) for polarization vectors lying in the polarization plane $e_1'e_2'$ gives:

$$\mathrm{w}(e)=2\left(\left|\left(p_ie^*\right)L_l+m\left(e_1'e^*\right)\eta D_+\right|^2-\beta_{res}l(kk_f)L_l^2+\right.$$

$$\left.+\frac{m^2\eta^2}{4}\frac{(kk_f)^2}{(kp_i)(kp_f)}\left(D_+^2-D_2L_l\right)\right), \qquad (24)$$

$$D_2=2L_l+\left(L_{l+2}+L_{l-2}\right). \qquad (25)$$

In the case when the polarization of the photon state lies on the plane $e_1'e_2'$:

$$\frac{W_1-W_2}{2m^2}=-\left(1+2\tau^2+\left(\frac{\tilde m}{m}\right)^2\left(\frac{1}{\rho}+\upsilon'_l\right)\beta_{\text{res}}\right)L_l^2+$$
$$+\eta^2\left(D_+^2-D_2L_l\right),\qquad(26)$$

$$\tau^2=\frac{-g^2}{m^2\beta_-^2}\sin^2\psi=\left(\frac{\tilde m}{m}\right)^2\frac{\left(1-\rho-\beta_{\text{res}}\right)}{\rho}\sin^2\psi,\qquad(27)$$

$$\frac{\mathrm{W}_{\text{unpolar}}}{2m^2}=-\left(1+2\upsilon'_l\frac{\tilde m^2}{m^2}\beta_{\text{res}}\right)L_l^2+$$
$$+\eta^2\left(1+2\upsilon'_l\rho\right)\left(D_+^2-D_2L_l\right),\qquad(28)$$

$$\frac{W_{\pi/4}^{(1,2)}-W_{-\pi/4}^{(1,2)}}{2m^2}=2\frac{\tilde m}{m}\sqrt{\frac{1-\rho-\beta_{\text{res}}}{\rho}}\sin\psi\times$$
$$\times\left(\eta D_++\frac{\tilde m}{m}L_l\sqrt{\frac{1-\rho-\beta_{\text{res}}}{\rho}}\cos\psi\right)L_l\propto\sin\psi,\qquad(29)$$

$$W_+^{(1,2)}-W_-^{(1,2)}=0.\qquad(30)$$

For planes $e'_1e'_3$ and $e'_2e'_3$, calculations (23) must be carried out directly. For polarization vectors lying in the polarization plane $e'_1e'_3$ we obtain:

$$\frac{W_1^{(1,3)}-W_3^{(1,3)}}{2m^2}=-\left(1+\tau^2-\left(\frac{\tilde m}{m}\right)^2\left(\frac{2}{\rho}+\upsilon'_l\right)\right)L_l^2+$$
$$+\eta^2\left(1+\upsilon'_l\,\rho\right)\left(D_+^2-D_2L_l\right),\qquad(31)$$

$$\frac{W_{\pi/4}^{(1,3)}-W_{\pi/4}^{(1,3)}}{2m^2}=\lambda\left(\frac{\tilde m}{m}\right)^2\sqrt{\frac{\left|\beta_{\text{res}}\right|}{\upsilon'_l}}\frac{\left(u_l+u'_l\right)}{2}\times$$
$$\times\left(L_l^2\sqrt{\frac{1-\rho-\beta_{\text{res}}}{\rho}}\cos\psi+\frac{m}{\tilde m}\eta L_lD_+\right),\qquad(32)$$

$$W_+^{(1,3)}-W_-^{(1,3)}=0,\qquad(33)$$

$$u_l=\frac{2l\left(kp_i\right)}{\tilde m^2},\quad u'_l=\frac{2l\left(kp_f\right)}{\tilde m^2}.\qquad(34)$$

For polarization vectors lying in the polarization plane $e'_2e'_3$ we obtain:

$$\frac{W_2^{(2,3)}-W_3^{(2,3)}}{2m^2}=\left(\tau^2-\left(\frac{\tilde m}{m}\right)^2\frac{\beta_{\text{res}}}{\rho}\right)L_l^2$$
$$+\eta^2\upsilon'_l\rho\left(D_+^2-D_2L_l\right),\qquad(35)$$

$$\frac{W_{\pi/4}^{(2,3)}-W_{\pi/4}^{(2,3)}}{2m^2}=\lambda\left(\frac{\tilde m}{m}\right)^2\sqrt{\frac{\left|\beta_{\text{res}}\right|}{\upsilon'_l}\frac{\left(1-\rho-\beta_{\text{res}}\right)}{\rho}}\times$$
$$\times\frac{\left(u_l+u'_l\right)}{2}L_l^2\sin\psi\propto\sin\psi,\qquad(36)$$

$$W_+^{(2,3)}-W_-^{(2,3)}=0.\qquad(37)$$

Formulas (26)–(37) were obtained for the process of photon emission by an electron in an electromagnetic wave field, but with such detail that would allow to obtain the corresponding expressions for other processes with simple substitutions.

a) Photon emission by an electron during scattering in an electromagnetic wave field.

Using the conservation laws for the parameters determining the probabilities (26)–(37) we obtain:

$$\rho=\frac{u'}{u_l},\quad u'=\frac{\left(kk_f\right)}{\left(kp_f\right)},\ 0\le u'\le u_l,\qquad(38)$$

$$\upsilon'_l=\frac{u_l}{4}\frac{u'}{1+u'},\quad u'_l=\frac{u_l}{1+u'}.\qquad(39)$$

b) Annihilation of an electron-positron pair in the field of a monochromatic electromagnetic wave.

It is necessary to perform substitutions: $p_f\to-\bar p_i$.

The differential probability has the form:

$$d\mathrm{w}_{fi}^{(l)}=-\mathrm{w}_0\mathrm{W}\delta^{(4)}\left(\tilde p_i+\tilde{\bar p}_i-|l|k-k_f\right)d^3k_f,\qquad(40)$$

where $l\le1$, parameters determining W (22) are obtained by:

$$\rho=\frac{\upsilon}{\upsilon_l},\quad\upsilon=\frac{\left(\left(kp_i\right)+\left(k\bar p_i\right)\right)}{4\left(kp_i\right)\left(k\bar p_i\right)},\qquad(41)$$

$$\upsilon_l=\frac{u_l+\bar u_l}{4},\quad\bar u_l=\frac{2l\left(k\bar p_i\right)}{\tilde m^2}.\qquad(42)$$

c) Formation of an electron-positron pair by a photon in the field of a monochromatic electromagnetic wave.

It is necessary to perform substitutions:

$$p_i\to-\bar p_f,\ k_f\to-k_i.$$

The differential probability has the form:

$$d\mathrm{w}_{fi}^{(l)}=-\mathrm{w}_0\mathrm{W}\delta^{(4)}\left(k_i+lk-\tilde p_f-\tilde{\bar p}_f\right)\times$$
$$\times d^3p_fd^3\bar p_f,\qquad(43)$$

where $l\ge1$, parameters determining W (22) are obtained by:

$$\rho=\frac{\upsilon'}{\upsilon_l},\quad\upsilon'=\frac{\left(kk_i\right)^2}{4\left(kp_f\right)\left(k\bar p_f\right)},\qquad(44)$$

$$\upsilon'_l\to\upsilon_l=\frac{l\left(kk_i\right)}{2\tilde m^2},\ 1\le\upsilon'\le\upsilon_l,\qquad(45)$$

$$u_l\to-\bar u'_l,\bar u'_l=\frac{2l\left(k\bar p_f\right)}{\tilde m^2}=\frac{\upsilon_l}{2}\left(1\pm\sqrt{1-1/\upsilon'}\right),\qquad(46)$$

$$u'_l=\frac{\upsilon_l}{2}\left(1\mp\sqrt{1-1/\upsilon'}\right),\qquad(47)$$

d) Absorption of a photon by an electron in the field of a monochromatic electromagnetic wave.

It is necessary to perform substitutions: $k_f\to-k_i$

The differential probability has the form:

$$d\mathrm{w}_{fi}^{(l)}=-\mathrm{w}_0\mathrm{W}\delta^{(4)}\left(\tilde p_i+k_i-|l|k-\tilde p_f\right)d^3p_f\qquad(48)$$

CONCLUSIONS

In order to construct an analytical theory of 2nd-order processes with respect to the fine structure constant, the probabilities of 1st-order processes with respect to the fine structure constant for polarized photon states lying outside the mass surface were obtained. $k_f^2 \neq 0$.

In contrast to the case were the electron lies on a bulk surface, the probability contains additional terms related to both the additional polarization along the vector $e_3^{'}$, and the polarization effects of polarizations at angles $\pm 45^{\mathrm{o}}$ and circular polarizations in the planes, which forms the vector with $e_3^{'}$, $e_1^{'}$, $e_2^{'}$. In total, these are 5 additional terms compared to the case of a real photon [2].

In the limit $\beta_{res} = 0$ where the photon is on the mass surface, expression (20) becomes the well-known expression for the process of emission of a polarized photon by an electron in a wave field [2].

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