ELECTROMAGNETIC STRUCTURE OF LAMBDA Λ^0(1115) HYPERON IN CONSTITUENT QUARK MODEL

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Λ^0 (1115) is the next (after neutron) the neutral baryon and the first one which contains the hard s-quark. Therefore it's very interesting to analyse the sign and the value of Λ mean-square charge radius (MSCR) $< r_\Lambda^2 >$ and so to elucidate the natural questions: does the Λ charge structure reproduce the neutron phenomenon (zero or negative sign of $< r_n^2 >$) or not?

The previously published theoretical predictions for $< r_\Lambda^2 >$ are disposed in very wide spectrum and admit both positive and negative values of this static property [1]:

$$-0.28\, fm^2 \lesssim < r_\Lambda^2 > \lesssim +0.11\, fm^2. \quad (1)$$

Therefore, the new "obvious" calculations will be the fresh addition and seem to be useful for more clear understanding of realistic value of $< r_\Lambda^2 >$.

In this paper the results of new calculations of lambda Λ^0 (1115) MSCR $< r_\Lambda^2 >$ are presented. In nonrelativistic constituent quark model (NRCQM) [2-3], as a first step, only the spin-independent part of quark-quark interaction is included:

$$H_I = \frac{1}{2} K \sum_{i<j} \left(\vec{r_i} - \vec{r_j}\right)^2. \quad (2)$$

In harmonic oscillator model (HOM) the coupling constant K (i.e. the strength of confinement) is defined anew and may be expressed in terms of charge proton radius r_p and the mass m_u of u-quark:

$$K = \frac{1}{3m_u \cdot r_p^4}. \quad (3)$$

Let's underline here that this method of K-determination is essentially new element of the present approach to Λ structure.

Now let's make the transition from three general radius-vectors $\vec{r_1}, \vec{r_2}, \vec{r_3}$ in eq. (2) to the two vectors $\vec{\rho}, \vec{\lambda}$ of relative coordinates and centre-of-mass vector $\vec{R}$. The change of variables converts H_I from eq. (2) to the form

$$H_I = \frac{3}{2} K \overrightarrow{\rho^2} + \frac{3}{2} K \overrightarrow{\lambda^2}. \quad (4)$$

In terms of HOM masses and frequencies for the case $m_1 = m_2 \equiv m$, $m_3 \neq m$ eq. (4) may be rewritten as

$$H_I = \frac{1}{2} m_\rho \omega_\rho^2 \cdot \overrightarrow{\rho^2} + \frac{1}{2} m_\lambda \omega_\lambda^2 \cdot \overrightarrow{\lambda^2}, \quad (5)$$

where $m_\rho = m$, $m_\rho \omega_\rho = (3K/m)^{1/2}$,

$$\omega_\rho^2 = 3K/m_\rho, \quad (6)$$

$$m_\lambda = \frac{3mm_3}{2m+m_3}, \ \omega_\lambda^2 = 3K/m_\lambda. \quad (7)$$

The eigenstates of H_I (5) are well known. The ground state wave function $\Psi(\vec{\rho},\vec{\lambda})$ has both (ρ, λ) oscillators in their respective ground states:

$$\Psi(\vec{\rho},\vec{\lambda}) = \left(\frac{m_\rho \omega_\rho}{\pi}\right)^{\frac{3}{4}} \left(\frac{m_\lambda \omega_\lambda}{\pi}\right)^{\frac{3}{4}} \times exp\left(-\frac{1}{2} m_\rho \omega_\rho \overrightarrow{\rho^2} - \frac{1}{2} m_\lambda \omega_\lambda \overrightarrow{\lambda^2}\right). \quad (8)$$

For $\Lambda\,(uds)$ hyperon the proper quarks masses are $m_1 = m_2 = m_u = m_d \equiv m$ and $m_3 = m_s$.The square charge radius r_Λ^2 is

$$r_\Lambda^2 = \sum_{i=u,d,s} e_i \left(\vec{r_i} - \vec{R}\right)^2, \quad (9)$$

where e_i are the electric charges of the proper three quarks. The result of calculations is the following:

$$r_\Lambda^2 = \frac{1}{6} \overrightarrow{\rho^2} + \frac{\sqrt{3} m_s}{(2m+m_s)} \cdot (\vec{\rho} \cdot \vec{\lambda}) - \frac{1}{2} \frac{(2m-m_s)}{(2m+m_s)} \cdot \overrightarrow{\lambda^2}. \quad (10)$$

By definition the Λ^0 (1115) MSCR $< r_\Lambda^2 >$ equals to

$$< r_\Lambda^2 > = \langle \Psi | \textstyle\sum_i e_i \left(\vec{r_i} - \vec{R}\right) | \Psi \rangle. \quad (11)$$

Averaging of eq. (11) with the help of wave function (8) amounts to the standard Gaussian integrations and the final results may be written as

$$\left\langle \Psi \left| \overrightarrow{\rho^2} \left(\overrightarrow{\lambda^2}\right) \right| \Psi \right\rangle = \frac{3}{2} \cdot \frac{1}{m_{\rho(\lambda)} \omega_{\rho(\lambda)}}, \quad (12)$$

$$\langle \Psi | \vec{\rho} \cdot \vec{\lambda} | \Psi \rangle = 0. \quad (13)$$

So for $< r_\Lambda^2 >$ we obtain the general result

$$\langle \Psi | r_\Lambda^2 | \Psi \rangle = \frac{1}{4} \cdot \frac{1}{(m_\rho \omega_\rho)} - \frac{3}{4} \frac{(2m-m_s)}{(2m+m_s)} \cdot \frac{1}{(m_\lambda \omega_\lambda)}, \quad (14)$$

where

$$m_\rho \omega_\rho = (3Km)^{1/2},$$

$$m_\lambda \omega_\lambda = 3 \left(K \frac{mm_s}{(2m+m_s)}\right)^{1/2}. \quad (15)$$

So, the new analytical representation for Λ MSCR in uds-basis may be written in the final form as

$$< r_\Lambda^2 > = \frac{1}{4} r_p^2 \left[1 - \frac{\sqrt{3}(2m-m_s)}{\sqrt{m_s(2m+m_s)}}\right]. \quad (16)$$

Note that in formal limit $m_s \to m$ eq. (24) is transformed into "correct neutron" limit $< r^2 > \to 0$.

For numerical calculations we assume the most commonly used constituent quark masses

$$m \equiv m_u = m_d = 0.33\, GeV, \quad (17)$$

$$m_s = 0.51\, GeV, \quad (18)$$

and the most recent proton charge radius

$$r_p \cong 0.84\, fm. \quad (19)$$

Taking together these numerical values, we obtain the following results:

$$< r_\Lambda^2 > = 0.117\, fm^2 \quad (\cong 0.12\, fm^2). \quad (20)$$

In other words, in NRCQM the Λ MSCR (i) is positive, (ii) small enough, and so (iii) the "neutron phenomenon" (theoretical zero or experimental negative value of $< r_n^2 >$) is not reproduced.

Experimental facilities: how can we measure the Λ MSCR? In principle four possibilities may be listed [4–7].

1. Elastic scattering of high-energy Λ -beam on atomic electrons of the proper target (e.g. for Be-target):

$$\Lambda^0 + e^- \to \Lambda^0 + e^-. \quad (21)$$

2. Electroproduction of strange particles on the proton target:

$$e^- + p \to e^- + \Lambda^0 + K^+. \quad (22)$$

3. Production of polarized Λ hyperon-antihyperon pair in electron-positron colliders:

$$e^+ + e^- \to \Lambda^0 + \overline{\Lambda}^0. \quad (23)$$

4. Dalitz decay $\psi(2S) \to \Lambda\overline{\Lambda}e^+e^-$ in time-like region with

$$q^2 \gtrsim 4m_e^2 = 1.05 \times 10^{-6}\ GeV^2. \quad (24)$$

Now about the $\Lambda^0(uds)$ magnetic moment (MM). Experimental (exp) value of Λ MM now is well established:

$$\mu_{exp}(\Lambda^0) = -0.613 \pm 0.004\ \mu_N, \quad (25)$$

where as usual, $\mu_N = e/2m_p$ is nuclear magneton (n.m). Usually this value is used for determination of the mass of strange s-quark:

$$m_s = 0.51\ GeV. \quad (26)$$

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