INFLUENCE OF MULTIPLE SCATTERING ON BREMSSTRAHLUNG BY RELATIVISTIC ELECTRONS IN A THIN ORIENTED CRYSTAL

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The effect of multiple scattering of ultrarelativistic electrons on the lattice atoms of a thin oriented crystal is investigated. It is shown that when the condition of dipole mode of radiation is violated, a radiation suppression effect should take place. This effect is similar to the Ternovskii-Shul'ga-Fomin effect of suppression of radiation in a thin amorphous target, but in the case of a thin oriented crystal the coherent bremsstrahlung is suppressed. The possibility and conditions for experimental observation of this effect at the DESY accelerator is discussed.

PACS: 34.80.Dp, 61.85.+p

INTRODUCTION

Landau and Pomeranchuk showed [1] that the multiple scattering of ultrarelativistic electrons on the atoms of matter can lead to the suppression of bremsstrahlung compared to the predictions of the Bethe-Heitler theory [2]. This occurs when the RMS angle of multiple scattering of electronon atoms within the so-called radiation formation zone or coherence length [3] exceeds the characteristic radiation angle of the relativistic electron $1/\gamma$, where γ is the electron Lorenz-factor. A quantitative theory of this effect was later developed by Migdal [4], and now this effect is called the Landau-Pomeranchuk-Migdal effect (or the LPM effect) [5].

The experimental verification of the Migdal's theory was carried out at SLAC for the electron beam energies up to 25 GeV using a wide range of target thicknesses and kind of materials: from aluminum to uranium [6, 7]. The results of this study generally confirmed the predictions of the Migdal theory of the LPM effect [4]. However, in the case of relatively thin targets, there were discrepancies with the Migdal's theory.

In [8] it was shown that these discrepancies in the SLAC experiment were connected with the effect of suppression of bremsstrahlung in a thin layer of matter theoretically predicted earlier in [9].

Although both effects are of a similar nature, they nevertheless give significantly different predictions in the behavior of the spectral density of radiation depending on the energy of the emitted gamma quanta and, especially, depending on the target thickness.

A quantitative theory of the effect of suppression of radiation in a thin layer of matter was developed in [10, 11]. A detailed experimental study of this effect was carried out by the NA63 collaboration at the CERN SPS accelerator at electron energies up to 250 GeV [12, 13]. After the experimental confirmation of the main predictions of the theory [8-11], this effect was named the Ternovskii-Shul'ga-Fomin effect (or TSF effect) after the authors of the theoretical prediction.

Analogues of LPM and TSF effects should take place when relativistic electrons pass through oriented crystals [5, 8, 14, 15], but in this case, coherent bremsstrahlung radiation will be suppressed. In the CERN experiment to study channeling radiation [16], the LPM effect was observed in a relatively thick crystal. An experimental study of the TSF effect in oriented crystals has not yet been carried out.

In this paper, the possibility of setting up an experiment to study the TSF effect in thin crystals at the DESY accelerator is studied. Preliminary consideration shows that the presence of a coherent effect in the scattering and radiation of relativistic electrons in the crystal makes it possible to observe the TSF effect at electron energies up to 6 GeV at the DESY accelerator.

1. EFFECT OCCURRENCE CONDITIONS

The main condition for the occurrence of both the LPM and TSF effects is the violation of the dipole regime of radiation, that is, the excess of the RMS angle of multiple scattering of electron within the coherence length l_c over the characteristic radiation angle of the relativistic electron $\theta_k \sim 1/\gamma$, i.e.

$$\gamma\theta_{ms}(l_c) > 1. \tag{1}$$

The coherence length of the bremsstrahlung increases rapidly with the electron energy increase and decrease in the energy of the emitted gamma quantum [2]:

$$l_c \approx 2\varepsilon\varepsilon'/m^2\omega\,, \tag{2}$$

where ε and ε' are the electron energy before and after radiation, m is the electron mass, ω is the energy of the emitted gamma quantum, $\varepsilon = \varepsilon' + \omega$ ($c = \hbar = 1$).

The difference between the LPM and TSF effects is the conditions of their observation. Thus, Migdal's theory of the LPM effect [4] describes radiation in a boundless amorphous medium, and therefore is suitable for describing the effect of multiple scattering on bremsstrahlung in relatively thick targets, when $T > l_c$. To describe radiation in a thin target $T < l_c$ the approach developed in [10,11] should be used.

For the TSF effect to occur, two conditions must be met simultaneously: the first, a "thin target" $T < l_c$, and the second, a non-dipole radiation regime (1), i.e.

$$l_c > T > l_\gamma. \tag{3}$$

Resolving the left side of this inequality with respect to the energy of the emitted gamma-quantum using expressions (2), we obtain

$$\omega < \omega_{TSF} \equiv 2\gamma^2/T, \qquad (4)$$

where ω_{TSF} is the boundary energy of the gamma quantum, separating the regions of the spectrum of bremsstrahlung, in which two effects of radiation suppression are realized: TSF (at $\omega < \omega_{TSF}$) and LPM (at $\omega > \omega_{TSF}$). Recall that in both of these cases we are talking about a non-dipole radiation regime when condition (1) is satisfied, which in terms of the energy of gamma quanta can be represented as

$$\omega < \omega_{LPM} \equiv \frac{32\pi Z^2 e^4 n}{m^4}\varepsilon^2 \ln(ma_{TF}). \qquad (5)$$

In this case, ω_{LPM} is the boundary energy of the gamma quantum, below which the non-dipole mode of radiation is realized, and either both effects of radiation suppression of LPM and TSF can take place. The contribution of so-called dielectric suppression or Ter-Mikaelian effect [3] is dominant in the soft part of spectrum at

$$\omega < \gamma\omega_p. \qquad (6)$$

When preparing an experiment to test the TSF effect in a crystal, it is necessary to select such a region of the spectrum of emitted gamma quanta in which this effect could be observed most clearly and without interference with other mechanisms of radiation suppression. Simultaneous consideration of conditions (4) and (6) determines the optimal region of the radiation spectrum for observing the TSF effect as:

$$\gamma\omega_p < \omega < 2\gamma^2/T\ . \qquad (7)$$

At the same time, the condition of non-dipole radiation must also be satisfied, i.e. the inequality (1), in which instead of θ_{ms} the RMS angle of coherent scattering of the particle in the crystal θ_{cr} should be used, and the coherence length l_c must be replaced by the crystal thickness T_{cr}, since according to condition (3) the thickness of the target must be less than the coherence length. Thus, for radiation in a thin oriented crystal, the non-dipole condition has the form:

$$\gamma\theta_{cr}(T_{cr}) > 1. \qquad (8)$$

For example, in the case of using a silicon crystal with the thickness of $T_{cr} = 10$ μm oriented along the axis <111> to the electron beam with the energy ε=6 GeV condition (7) gives:

$$0.3 \text{ MeV} < \omega < 4 \text{ MeV}. \qquad (9)$$

2. COHERENT EFFECT IN SCATTERING OF FAST ELECTRON IN CRYSTAL

Unlike an amorphous substance, atoms in a crystal are arranged strictly periodically. If a relativistic charged particle moves along one of the crystal axes, then there are significant correlations in the successive interactions of such a particle with atoms located along this axis. These correlations lead to significant differences in electrodynamic processes in crystals compared to the processes in an amorphous medium.

In elastic scattering, these correlations are manifested in the occurrence of a coherent effect in the scattering of a particle, when all atoms of a given string or plane of the crystal scatter the incoming particle in the same direction.

The coherent effect in scattering occurs when fast charged particles move at a small angle $\psi \sim \psi_L$ regarding one of the crystal axes, where ψ_L is the Lindhard angle [17]:

$$\psi_L = \sqrt{4Ze^2/\varepsilon d}, \qquad (10)$$

where d is the distance between atoms along the crystal axis, $Z|e|$ is the charge of an atomic nucleus. This effect manifests itself in intense scattering along the azimuthal angle φ (Fig. 1) and also known as the «doughnut scattering effect» (Fig. 2).

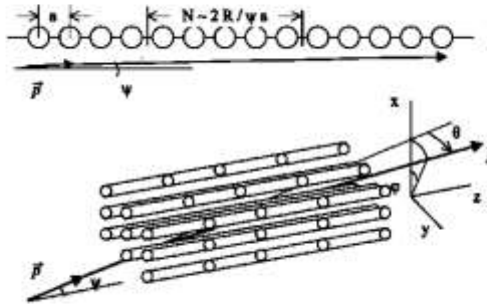

Fig. 1. Scattering of fast charged particles on of crystal atomic strings

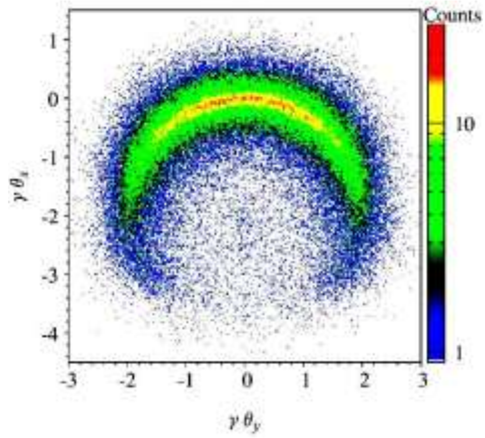

Fig. 2. The angular distribution of scattered 6 GeV electrons impinged the Si crystal of 10 μm thickness at ψ_L angle to the axis ⟨111⟩. Results of computer simulation. The colors show the density distribution of particle scattering angles

According to the theory of multiple scattering on chains of crystal atoms, developed in [18], the mean square angle of multiple scattering of electron (positron) on chains of crystal atoms $\langle\theta_{cr}^2\rangle$ exceeds the corresponding mean square of the scattering angle in an amorphous medium of the same thickness $\langle\theta_{ms}{}^2\rangle$ by a factor

$$\frac{\langle\theta_{cr}^2\rangle}{\langle\theta_{ms}^2\rangle} \approx \frac{R}{8\psi d}, \qquad (11)$$

where R is the atomic screening radius [5].

This means that conditions for observing the TSF effect can be realized in a crystal at significantly lower electron energies than in an amorphous target.

3. TSF EFFECT IN A THIN CRYSTAL

The spectral density of bremsstrahlung radiation in a thin layer of matter ($T \ll l_c$), defined by the formula [30]:

$$\frac{dE_{TSF}}{d\omega}=\frac{2e^2}{\pi}\int d\theta_s\ f_{BM}(\theta_s)\left[\frac{2\xi^2+1}{\xi\sqrt{\xi^2+1}}\ln(\xi+\sqrt{\xi^2+1})-1\right],\quad \xi=\gamma\theta_s/2. \quad (12)$$

Note that the spectral density of radiation in the thin target (12) does not depend on the energy of the emitted gamma quantum and is completely determined by the parameter ξ, which is the non-dipole parameter of radiation (1) multiplied by 1/2.

Substituting into (12) the distribution function $f_{BM}(\theta_s)$ on electron scattering angle θ_s, obtained as a result of computer simulation of the scattering of electrons with an energy of 6 GeV at various angles of incidence relative to the <111> axis of a silicon crystal 10 µm thick, we obtain the orientational dependence of the spectral density of radiation for gamma quanta in the energy range (9), shown in Fig. 3 by the solid red line.

The green doted curve "T-M" is the result of calculations according to the formulas of the Ter-Mikaelian theory of coherent bremsstrahlung in a crystal [3], based on the use of the first Born approximation. In this case, there is a significant increase in the intensity of the bremsstrahlung radiation in the crystal compared to the radiation in an amorphous target of the same thickness shown in Fig. 3 with the green dot-dashed line "B-H" [2]. This excess is approximately $R/8\psi d$ times as in (11).

The use of the first Born approximation actually means neglecting the curvature of the particle's trajectory in the field of the crystal lattice, which is acceptable when the relativistic electron motion is far from the channeling regime, i.e. when $\psi \gg \psi_L$. Consideration of trajectory curvature when $\psi \sim \psi_L$ in the formula (12) at the dipole approximation makes it possible to eliminate the formal infinity for coherent bremsstrahlung radiation at $\psi \to 0$, however, it gives an overestimated result (see the blue dashed curve "Dipole" in Fig. 3) compared to the result of the theory of the TSF effect [10, 11] (red solid curve) that demonstrate the suppression of radiation.

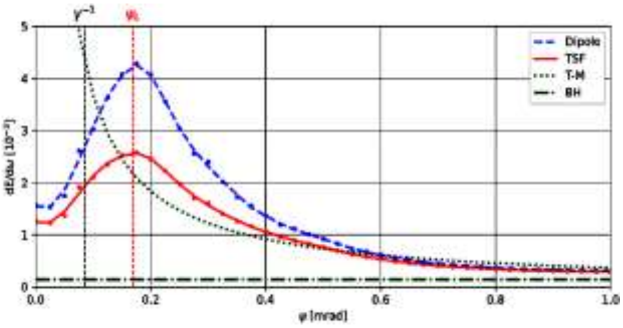

Fig. 3. Orientation dependence of the spectral density of coherent radiation of 6 GeV electrons in 10 µm thicksilicon crystal when the beam falls at an angle ψ regarding the axis <111> (Details in the text)

ACKNOWLEDGEMENTS

The work was partially supported by the project No. 531314364 of the German Research Foundation (STCU project No. P811).

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