

# TROPHIC MODELS OF THE EARTH'S HEAT BALANCE

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The current state of observations regarding the preservation of the Earth's thermal balance (ETB) is considered, where the emergence of the Earth's energy imbalance (EEI) is discussed. A model describing the conservation of ETB based on the determination of the main factors of the process in the absence of external influences has been proposed. This model is based on the model of interaction between two species "predator-prey" type. Periodic changes in the heat flux of absorption/emission by the atmosphere and the Earth's surface are observed in this model. In the presence of external disturbing factors, it is proposed to analyze small deviations of the determining parameters of the ETB using the Rosenzweig-MacArthur tritrophic model "prey-predator-superpredator". The use of the tritrophic model is based on the analogy of the trophic chain with a chain consisting of radiation from the Earth's atmosphere, absorption by the Earth's surface, and the influence of external factors. A qualitative description of the ETB for the tritrophic model under Shilnikov chaos conditions is obtained. The proposed trophic models are not indisputable, but they have all the features that allow us to consider them as providing a qualitative description of the ETB, considering the experimentally observed EEI of the Earth.

## INTRODUCTION

It is known that the Sun-Earth-Space system maintains a nearly constant Earth thermal balance (ETB) throughout its existence. This means that, over the long term, the amount of heat emitted by the Sun and the amount of heat received from the Sun must equal the heat radiated by the Earth into space.

When considering the TBC, energy flows averaged over time and over the Earth's surface, taking into account the atmosphere, through a unit area are usually used, neglecting the thickness of the atmosphere compared to the radius of the Earth.

For example, with this description, the greatest influx of energy to the Earth is provided by solar radiation in the spectral range from 0.1 to 4  $\mu\text{m}$  (shortwave radiation – SWR) and amounts to an average value over the entire surface of the planet of about  $341 \text{ W}\cdot\text{m}^{-2}$ . There is also a small influx of energy from internal heat sources (radioactive decay, density stratification) in comparison with the solar one, amounting to about  $0.08 \text{ W}\cdot\text{m}^{-2}$ , which is not shown in the figure [1]. This influx of energy from internal heat sources is not taken into account below due to its smallness.

Of the  $341 \text{ W}\cdot\text{m}^{-2}$  of solar radiation falling on Earth,  $102 \text{ W}\cdot\text{m}^{-2}$  is reflected back into space ( $23 \text{ W}\cdot\text{m}^{-2}$  from the Earth's surface +  $79 \text{ W}\cdot\text{m}^{-2}$  from clouds). This portion of solar radiation is reflected back into space by clouds, atmospheric molecules, tiny suspended particles called aerosols, as well as the Earth's surface, snow, and ice.

Of the  $161 \text{ W}\cdot\text{m}^{-2}$  of energy absorbed by the Earth's surface,  $40 \text{ W}\cdot\text{m}^{-2}$  returns to space in the form of thermal radiant electromagnetic radiation in the wavelength range from 3 to 45  $\mu\text{m}$  (long-wave radiation – LWR). Another  $97 \text{ W}\cdot\text{m}^{-2}$  are transferred to the atmosphere due to various thermal processes ( $80 \text{ W}\cdot\text{m}^{-2}$  – water evaporation +  $17 \text{ W}\cdot\text{m}^{-2}$  – convective heat exchange). In addition, about  $356 \text{ W}\cdot\text{m}^{-2}$  of the Earth's radiation is absorbed by the atmosphere, of which  $332 \text{ W}\cdot\text{m}^{-2}$  ( $161 - 40 - 97 - 356 + 332 = 0$ ) returns as atmospheric backradiation.

Thus, the total thermal radiation of the Earth's surface is  $396 \text{ W}\cdot\text{m}^{-2}$  ( $356 + 40$ ), which, according to the Stefan-

Boltzmann law, corresponds to an average thermal temperature of 288 K ( $15^\circ\text{C}$ ) [1].

A more thorough analysis of the Earth's average energy balance for 2000–2024 has now revealed a warming imbalance. This imbalance is due to more precise measurements of its dynamics using radiometric observations at the top of the atmosphere and a more accurate analysis of the increase in ocean heat content [2].

Satellite observations of clouds and the Earth's radiant energy balance indicate that the Earth's positive energy imbalance has doubled from  $(0.5 \pm 0.2) \text{ W}\cdot\text{m}^{-2}$  during the first 10 years of this century to  $(1.0 \pm 0.2) \text{ W}\cdot\text{m}^{-2}$  during the last decade [3]. The increase in the imbalance is the result of an increase in absorbed solar radiation by  $(0.9 \pm 0.3) \text{ W}\cdot\text{m}^{-2}$ , which is partially offset by an increase in outgoing longwave radiation by  $(0.4 \pm 0.25) \text{ W}\cdot\text{m}^{-2}$ . Overall, it is argued that a steady acceleration of climate warming has been observed over the period under consideration. The role of cloud cover in the Earth's thermal energy balance is highlighted in [3].

In the eighties of the last century, heat flows Fig. 1 had different meanings [4, 5]. For example, the equations of the heat balance of the Earth's surface (1) and the heat balance of the atmosphere (2) can be represented as:

$$(E_2 - E_3) - (F_2 - F_1) = 105 \text{ W}\cdot\text{m}^{-2}, \quad (1)$$

$$(E_1 - E_4) + (F_3 - F_1) = 80 \text{ W}\cdot\text{m}^{-2}, \quad (2)$$

where  $(E_2 - E_3) = 157 \text{ W}\cdot\text{m}^{-2}$  – part of the SWR absorbed by the Earth,  $F_1 \approx F_3 \approx 343 \text{ W}\cdot\text{m}^{-2}$  – the part of the thermal LWR of the atmosphere's counter-radiation absorbed by the Earth's surface,  $F_2 = 392 \text{ W}\cdot\text{m}^{-2}$  – thermal radiation of the Earth's surface,  $(E_1 - E_4) = 80 \text{ W}\cdot\text{m}^{-2}$  – part of the SWR absorbed by clouds, aerosols and atmospheric gases.

From the magnitude of thermal radiation of the Earth's surface, it follows that the average annual temperature of the Earth's surface is slightly lower than [1], equal to  $+14.2^\circ\text{C}$ .

The average annual ETB is important because it allows us to determine the dynamics of changes in Earth's surface temperature.

The aim of this study is to analyze shorter-term changes in ETB, as they can have a significant impact on long-term ETB forecasts due to their cumulative effect.

For the Earth's thermal system, the equilibrium state (1), (2) is valid, where the equilibrium values of the determining parameters can be represented as:  $E_i = E_{i0}$ ,  $i = 1, 2, 3, 4$ ;  $F_k = F_{k0}$ ,  $k = 1, 2, 3$ .

Deviations of defining parameters  $E_i$  and  $F_i$  from the equilibrium we will represent in the form:  $E_i = E_{i0} + \delta E_i$ ,  $F_k = F_{k0} + \delta F_k$ .

In this case, the change in equilibrium (1), (2) over time can be described by a system of equations:

$$\frac{dx}{dt} = ax - bxy, \quad (3)$$

$$\frac{dy}{dt} = -dy + cxy, \quad (4)$$

where  $(E_{20} - E_{30}) - (F_{20} - F_{10}) = 105 \text{ W} \cdot \text{m}^{-2}$ ,  $(E_{10} - E_{40}) - (F_{30} - F_{10}) = 80 \text{ W} \cdot \text{m}^{-2}$ ,  $F_{10} = F_{30} = 343 \text{ W} \cdot \text{m}^{-2}$ ,  $F_{20} = 392 \text{ W} \cdot \text{m}^{-2}$ ,  $\delta E_1 - \delta E_4 = x$  - deviation of the absorption of the SWR by the atmosphere from the equilibrium,  $\delta F_3 - \delta F_1 = x$  - deviation of radiation by the atmosphere from equilibrium,  $\delta E_2 - \delta E_3 = y$  - deviation of the absorption of the SWR by the Earth's surface from equilibrium,  $\delta F_2 - \delta F_1 = y$  - deviation of the effective radiation of the Earth's surface from equilibrium.

The description of the temporal dynamics of the heat balance between the Earth and the atmosphere by equations (3), (4) is analogous to the Lotka-Volterra "prey-predator" model [6, 7]. This model describes the dynamics of the interaction of two types of biological systems, in which one species is considered a predator and the other as a prey. With regard to heat exchange between the Earth and the atmosphere (3), (4), the heat flow from the Earth's surface  $x$  can be considered as the number of prey, and the absorption of the heat flow by the atmosphere  $y$  can be considered as the number of predators.

In the problem under consideration, the model of interaction between prey and predator can be simplified and represented as a sequence of the following processes:

- process *a*) - in a cloudless atmosphere and in the absence of aerosols, solar radiation hits the Earth's surface, warms it, and leads to the evaporation of water and the formation of clouds and aerosols in the atmosphere, which can then be considered predators. The prey in this case is the flux of solar radiation to the Earth's surface;

- process *b*) - the formation of clouds and aerosols in the atmosphere leads to a decrease in the flux of solar radiation to the Earth's surface, cooling it and causing the disappearance of clouds and aerosols, which returns the heat exchange process between the Earth and the atmosphere to event *a*), repeating it again.

In equations (3) and (4), parameters  $a$  and  $b$  describe the maximum rate of increase of the power flux to the Earth's surface and the influence of the atmosphere on the rate of decrease of the power flux to the Earth's surface, respectively. Parameters  $d$  and  $c$  describe the rate of decrease of the atmospheric power flux and the influence of the power flux to the Earth's surface on the rate of increase of the atmospheric power flux, respectively.

All parameters in equations (3) and (4) are real and positive numbers.

## ANALYSIS OF SMALL DEVIATIONS OF THE ETB IN THE ABSENCE OF EXTERNAL DISTURBING FACTORS

To analyze the stability of the ETB, we determine the possible values of the constants of the system of equations (5), (6) and the initial values  $x(0)$  and  $y(0)$ . In this case, we will be guided by experimental data from long-term observations of the ETB [2, 3].

Periodic changes in heat fluxes can be observed in the absence of external factors (processes such as transition-to-El Niño (2010–2016) or post-El Niño (2016–2022) or various cyclones). However, in real conditions, these disturbing processes are always present and, in the language of the prey–predator model, a third participant in the trophic chain, called a "superpredator," comes into play. In our consideration, the "superpredator" is the above-mentioned processes such as transition-to-El Niño (2010–2016) or post-El Niño (2016–2022) or various cyclones. This combination of the above-mentioned processes forms the Rosenzweig–MacArthur tritrophic model "prey–predator–superpredator," which is studied using the "channels and jokers" method in trophic chains [8, 9] or in chemical reactions.

## ANALYSIS OF DEVIATION OF THE TBZ UNDER THE INFLUENCE OF EXTERNAL FACTORS

As a result of statistical analysis of the data from work [3], new dependences of Earth's EEI on time for dimensionless outgoing long-wave radiation of the atmosphere were obtained  $\bar{X}(t)$  and solar radiation absorbed by the Earth's surface  $\bar{Y}(t)$ :

$$\begin{aligned} \bar{X}(t) &= 0.287 - 0.021 \cdot t \pm 0.267, \\ \bar{Y}(t) &= -0.558 + 0.051 \cdot t \pm 0.203. \end{aligned} \quad (5)$$

The influence of external factors such as transition-to-El Niño (2010–2016), post-El Niño (2016–2022) or various cyclones will be described by the effect of a "superpredator" and determined by the quantity  $Z(\tau)$ . As a result of the impact of the "superpredator" in the ETB, deviations from the equilibrium value of outgoing long-wave radiation of the atmosphere may occur  $X(\tau)$  and solar radiation absorbed by the Earth's surface  $Y(\tau)$ .

We will describe such deviations using the Rosenzweig-MacArthur system "prey – predator – superpredator":

$$\frac{dX(\tau)}{d\tau} = \frac{X(\tau)}{\xi} \left( 1 - X(\tau) - \frac{Y(\tau)}{\beta_1 + X(\tau)} \right), \quad (6)$$

$$\frac{dY(\tau)}{d\tau} = Y(\tau) \left( \frac{X(\tau)}{\beta_1 + X(\tau)} - \delta_1 - \frac{Z(\tau)}{\beta_2 + Y(\tau)} \right), \quad (7)$$

$$\frac{dZ(\tau)}{d\tau} = \epsilon Z(\tau) \left( \frac{Y(\tau)}{\beta_2 + Y(\tau)} - \delta_2 \right), \quad (8)$$

where  $X(\tau), Y(\tau); Z(\tau)$  – non-negative numbers;  $\tau$  – dimensionless time.

In the system (6) – (8) the conditions are assumed to be satisfied  $0 < \delta_{1,2} < 1$  and  $0 < \xi, \epsilon \ll 1$ .

Changes  $\bar{X}(t)$  and  $\bar{Y}(t)$  are slow enough and can be taken into account with the corresponding faster changes ETB  $X(\tau), Y(\tau), Z(\tau)$  as follows. For this, we assume that  $\tau$  proportionally  $t$ . This requirement follows from the

need for the number of maxima to match  $X(\tau)$  experimental data [3].

Because slow changes  $\bar{X}(t), \bar{Y}(t), \bar{Z}(t)$  are linear functions in time, then it is sufficient to divide the interval of solution of the equations (6) – (8) into two:  $0 \leq \tau \leq \tau_1$  and  $\tau_1 \leq \tau \leq 2\tau_1$ . In this case, the initial values  $\bar{X}(0), \bar{Z}(0)$  we will set the first interval such that at the beginning of the second interval they are positive, i.e.  $\bar{X}(\tau_1), \bar{Z}(\tau_1) > 0$ , and ensured a continuous transition of solutions from the first interval to the second. For example,  $\bar{X}(t), \bar{Y}(t), \bar{Z}(t)$  can be specified by the following dependencies:

$$\begin{aligned}\bar{X}(t) &= \alpha - 0.021 \cdot t, \\ \bar{Y}(t) &= \beta + 0.051 \cdot t, \\ \bar{Z}(t) &= \gamma - 0.045 \cdot t,\end{aligned}\tag{9}$$

where  $\alpha = 0.232, \beta = 0.001, \gamma = 0.496$  – constants that ensure the continuity of the solutions of the first and second intervals. The values of these constants can be chosen such that they are ensured by constant values of the equilibrium determining parameters of the ETB.

The result of the above assumptions, taking into account the representations (9), is the conclusion about the additive representation of the corresponding slow  $\bar{X}(t), \bar{Y}(t), \bar{Z}(t)$  and fast  $X(\tau), Y(\tau), Z(\tau)$  changes in the parameters of the ETB.

Adding linear dependencies (5) to the corresponding solutions  $X(\tau), Y(\tau), Z(\tau)$  gives a dependence of the defining parameters of the TBZ on time that is qualitatively similar to those measured by observations [3].

## CONCLUSIONS

Using recent experimental data as an example, this paper examines the current state of the problem of preserving the ETB and detecting Earth's EEI. The article examines the main components of the ETB and their temporal dynamics over various observation time scales. A linear increase in the Earth's atmospheric temperature was observed over a 22-year interval, which, due to transition-to-El Niño (2010–2016) or post-El Niño (2016–2022) processes, is divided into two intervals with different growth rates. Based on identifying the main factors in the ETB process in the absence of the aforementioned external influencing processes, a model of interaction between two species, the “predator-prey” type, is proposed. The sequence of their interactions can be represented as follows:

1. In a cloudless atmosphere and in the absence of aerosols, solar radiation hits the Earth's surface, warms it, and leads to the evaporation of water and the formation of clouds and aerosols in the atmosphere, which can then be considered predators. The prey is the flux of solar radiation reaching the Earth's surface.

2. The formation of clouds and aerosols in the atmosphere reduces the flux of solar radiation reaching

the Earth's surface, cools it, and causes the clouds and aerosols to disappear, returning the heat exchange process between the Earth and the atmosphere to point 1, repeating itself. It is shown that in this case, periodic changes in heat fluxes are observed over time.

In the presence of external disturbing factors, an analysis of small deviations of the ETB is proposed based on the Rosenzweig-MacArthur tritrophic model “prey – predator – superpredator”.

This approach is not indisputable, but it has all the features that allow us to consider it as one that provides a qualitative representation of the defining parameters of the ETB, taking into account the experimentally observed Earth's EEI.

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## ТРОФНІ МОДЕЛІ ТЕПЛОВОГО БАЛАНСУ ЗЕМЛІ

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Розглянуто сучасний стан спостережень щодо збереження теплового балансу Землі (ТБЗ), де виявлено появу енергетичного дисбалансу (ЕД) Землі. За відсутності зовнішніх впливів, на основі визначення основних факторів процесу збереження ТБЗ запропоновано модель опису його існування, яка заснована на моделі взаємодії двох видів типу «хижак-жертва». Показано, що в цій моделі опису ТБЗ спостерігаються періодичні зміни теплового потоку поглинання/випромінювання атмосферою та поверхнею Землі. За наявності зовнішніх збуджуючих факторів пропонується аналізувати невеликі відхилення визначальних параметрів ТБЗ тритрофною моделлю Розенцвейга-Макартура «жертва-хижак-суперхижак». Використання тритрофної моделі базується на аналогії трофічного ланцюга з ланцюгом, яким є випромінювання земної атмосфери, поглинання поверхнею Землі та вплив зовнішніх факторів. Для тритрофної моделі в умовах шильніківського хаосу отримано якісний опис ТБЗ. Запропоновані трофні моделі не є безперечними, але вони мають всі ознаки, які дозволяють вважати їх такими, що забезпечують якісний опис ТБЗ, враховуючи експериментально спостережуваний ЕД Землі.