QUARK-DIQUARK MODEL OF BARYONS AND ELECTROMAGNETIC STRUCTURE OF Λ^0 (1115) HYPERON

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In the framework of quark-diquark model (QDM) the results of the calculations of Λ^0 (1115) hyperon mean-square charge radius (MSCR) are represented. Numerical values of $< r_\Lambda^2 >_{QDM}$ are crucially dependent on the mass m_{D_0} of scalar diquark $D_0 = (ud)$ $(J = 0)$ and therefore $\Lambda(QDM)$ MSCR may be both positive and negative $(-0.14\ fm^2 \leq\ < r_\Lambda^2 >_{QDM} \leq 0.20\ fm^2)$. But beside Λ^0, the scalar diquark D_0 also enters simultaneously in QD-strucutre of proton and neutron. So, the value $< r_\Lambda^2 >_{QDM} = +0.06\ fm^2$ with the $m_{D_0} = 0.434$ GeV is preferable due to demand of correct reproduction of experimental values of proton charge radius and neutron MSCR. Magnetic moment of $\Lambda(QDM)$ is also briefly discussed.

Due to M. Gell-Mann the conception of diquarks is as old as that of quarks (see the basic paper [1]). Soon after [1] the diquarks were concretized and developed in the papers [2–4]. Later such kind of structure as existence of relatively stable compact (q_1q_2)-object inside baryon was proposed in investigations of many inelastic reactions with proton. The application of quark-diquark model (QDM) to the description of the inner structure of separate nucleon is contained, for example, in refs. [5–8].

Here we'll try to do our own QDM-analysis of Λ^0 (1115) hyperon. We hope that such analysis may be the fresh addition to current understanding of Λ^0 which was achieved in standard nonrelativistic constituent quark model (NRCQM) [9–12].

In QDM it's supposed that inside baryon $B(q_1q_2q_3)$ two quarks q_1 and q_2 form diquark D – the (q_1q_2) compact bound state with definite mass and quantum numbers (so-called "inert core") and the almost all main properties of baryon are determined by the third quark (so-called "valence quark").

So, the starting point of QDM is the representation

$$B(q_1q_2q_3) = inert\ core + \\ + valence\ (v)\ quark = D(q_1q_2) + q_{3v}. \quad (1)$$

Generally speaking, the central idea and the evident advantage of QDM are the essential simplification of the whole formalism: two-body problem replaces the more complicated three-body problem.

Among other diquarks the scalar diquark $D_0 \equiv (ud)\, J = 0 \equiv (ud)_o$ with quantum number $I(J^\pi) = 0(0^+)$ plays exclusive role in the QD-structure of the first three baryons (p, n, Λ^0) in octet of the lightest baryons. So, the mass of D_0 is very important question. From the known literature's data the lower and the upper limits of $m(D_0)$ are the following:

$$0.30\ \text{GeV} \leq m(D_0) \leq 1.02\ \text{GeV}. \quad (2)$$

This interval is very large and is caused by the strong dependence of the diquark masses on the choice of q_1q_2-interaction potential inside diquark and the ways of accounting of kinetic and potential quark energies.

Hamiltonian of q_vD-system in harmonic oscillator model (HOM) in obvious notations has the form:

$$H = H_O + H_I = \frac{\vec{p}_v^{\,2}}{2m_v} + \frac{\vec{p}_D^{\,2}}{2m_D} + \\ + \frac{1}{2}K_D(\vec{r}_v - \vec{r}_D)^2. \quad (3)$$

Now let's introduce the relative coordinate $\vec{\rho}$ and center-of-mass coordinate $\vec{R}$:

$$\vec{\rho} = \vec{r}_v - \vec{r}_D,\ \vec{R} = \frac{m_v\vec{r}_v + m_D\vec{r}_D}{m_v + m_D}. \quad (4)$$

From eq.s (4) we obtain

$$\left.\begin{aligned} \vec{r}_v - \vec{R} &= \frac{m_D}{m_v + m_D}\cdot\vec{\rho}\,, \\ \vec{r}_D - \vec{R} &= -\frac{m_v}{m_v + m_D}\cdot\vec{\rho} \end{aligned}\right\}. \quad (5)$$

Due to eq.s (5) the H_I converts into

$$H_I = \frac{1}{2}K_D(\vec{r}_v - \vec{r}_D)^2 = \frac{1}{2}K_D\vec{\rho}^{\,2}. \quad (6)$$

Eq. (6) may be rewritten in oscillator form

$$\left.\begin{aligned} H_I &= \frac{1}{2}K_D\vec{\rho}^{\,2} = \frac{1}{2}m_\rho\omega_\rho^2\vec{\rho}^{\,2}\,, \\ m_\rho &= \frac{2m_vm_D}{m_v + m_D},\ \omega_\rho = \sqrt{K_D/m_\rho} \end{aligned}\right\}. \quad (7)$$

The wave function of ground state for Hamiltonian (7) equals to

$$\psi = \left(\frac{m_\rho\omega_\rho}{\pi}\right)^{\frac{3}{4}}\cdot e^{-\frac{1}{2}m_\rho\omega_\rho\vec{\rho}^{\,2}}. \quad (8)$$

By definition the Λ^0 (1115) square charge radius in QDM equals to

$$r_\Lambda^2 = \sum_{i=v,D_0} e_i\big(\vec{r}_i - \vec{R}_i\big)^2. \quad (9)$$

For Λ hyperon the valence quark is s-quark $\left(m_v = m_s, e_v = -\frac{1}{3}\right)$. Therefore, from eq.s (5) and (9) we obtain

$$r_\Lambda^2 = \frac{1}{3}\frac{(m_s - m_{D_0})}{(m_s + m_{D_0})}\cdot\vec{\rho}^{\,2}. \quad (10)$$

Averaging the value of $\vec{\rho}^2$ in eq. (10) by the wave function from eq. (8) amounts to the standard Gaussian integration ant the result in following:

$$\langle\psi|\vec{\rho}^2|\psi\rangle = \frac{3}{2}\frac{1}{m_\rho\omega_\rho}. \quad (11)$$

From eq.s (10) and (11) the Λ MSCR in QDM appears as

$$\left.\begin{aligned}\langle r_\Lambda^2\rangle_{QDM} &= \frac{1}{2}\frac{1}{\sqrt{K_D m_\rho}}\cdot\frac{(m_s - m_{D_0})}{(m_s + m_{D_0})},\\ m_\rho &= \frac{2m_s m_{D_0}}{m_s + m_{D_0}}\end{aligned}\right\}. \quad (12)$$

To complete the calculations, we propose to define the interaction constant K_D (i.e. the strength of confinement) in terms of m_s, m_{D_0}, well-known experimental (exp) value of charge proton radius r_p, and the mass of u-quark m_u. Really, from the proton QD-structure $p = (uud) = [u(ud)_0]$ and equality $\langle r_\mathrm{p}^2\rangle_{QDM} = \langle r_\mathrm{p}^2\rangle = \langle r_\mathrm{p}^2\rangle_{exp}$ we can calculate K_D and finally have

$$\langle r_\mathrm{p}^2\rangle_{QDM} = r_\mathrm{p}^2\frac{(m_s - m_{D_0})(m_u + m_{D_0})^2}{(m_s + m_{D_0})(m_u^2 + 2m_{D_0}^2)}\times$$

$$\times\sqrt{\frac{m_u(m_s + m_{D_0})}{m_s(m_u + m_{D_0})}}. \quad (13)$$

From eq. (13) it's clear that namely the factor $(m_s - m_{D_0})$ defines the "plus-minus" sign in $\langle r_\Lambda^2\rangle_{QDM}$-value in the whole interval in eq. (2).

For numerical values

$m_u = m_d = 0.33\ GeV,\ m_s = 0.51\ GeV, r_p = 0.84\ fm$ and m_{D_0} from eq. (13) we obtain the whole spectrum

$$-0.17\ fm^2 \le \langle r_\Lambda^2\rangle_{QDM} \le +0.23\ fm^2. \quad (14)$$

From eq. (14) we may conclude that (i) all values of $\langle r_\Lambda^2\rangle_{QDM}$ are relatively small and (ii) are oscillating near zero. But the whole interval nevertheless is wide enough. How can we select the preferable value of $\langle r_\Lambda^2\rangle_{QDM}$?

QD-structure of the neutron $n = n[(ud)_0 d]$ gives the answer together with well-known experimental value of $\langle r_\mathrm{n}^2\rangle_{exp}$:

$$\langle r_\mathrm{n}^2\rangle_{exp} = -0.1161\ fm^2 =$$

$$= \langle r_\mathrm{n}^2\rangle_{QDM} = r_\mathrm{p}^2\frac{(m_d^2 - m_{D_0}^2)}{(m_d^2 + 2m_{D_0}^2)}. \quad (15)$$

From eq. (15) we obtain

$$m_{D_0} = 0.434\ \text{GeV}, \quad (16)$$

and the final result for Λ MSCR in QDM quals to

$$\langle r_\Lambda^2\rangle_{QDM} = +0.06\ fm^2. \quad (17)$$

The result may surely be considered as a preferable realistic value of Λ MSCR in QDM.

Now about the $\Lambda^0(uds)$ magnetic moment (MM). The isoscalar nature of Λ^0 and s-quark admit only the presence of scalar diquark $(ud)_0$ with $I = J = 0$ inside Λ^0. Therefore the $\Lambda^0(uds)$ hyperon has the only QD-structure

$$\Lambda^0 = (ud)_0 + s \quad (18)$$

with valence s-quark. For such structure the equality

$$\mu(\Lambda^0) = \mu_s \quad (19)$$

is evidently correct. Therefore, the experimental value of Λ^0 MM$= -0.613\,\mu_N$ surely may be used as an input parameter for determination of the mass m_s of strange quark $m_s = 0.51$ GeV.

The main distinguishing features of Λ in QDM are the following:

- Λ^0 (1115)-hyperon may be successfully constructed using only scalar (ud)-diquark plus valence s-quark;
- All calculated values of $\langle r_\Lambda^2\rangle_{QDM}$ are comparatively small;
- These values are oscillating near zero in the interval

$$-0.17\ fm^2 \le \langle r_\Lambda^2\rangle_{QDM} \le +0.23\ fm^2$$

for all cited in the available literature masses of scalar (ud)-diquark (see eq. (2));

- The preferable value of $m(D_0) \equiv m((ud)_0)$ appears as

$$m(D_0) = 0.434\ \text{GeV}. \quad (20)$$

This value agrees with the QD-structure of the neutron and with the experimental value of neutron MSCR;

- Due to eq.s (13) and (20) the final result equals to $$\langle r_\Lambda^2\rangle_{QDM} = +0.06\ fm^2; \quad (21)$$
- It's especially interesting to underline that eq. (21) establishes the remarkable direct connection between the QDM and NRCQM including spin-dependent interaction (SDI) in the form of Fermi-Breit potential with renormalized coupling constant. The point is that earlier performed calculations of total $(uds + SDI)$ Λ^0MSCR [2] led to exactly the same as in eq. (21) result:

$$\langle r_\Lambda^2\rangle_{QDM} = \langle r_\Lambda^2\rangle_{uds+SDI} = $$
$$= 0.06\ fm^2! \quad (22)$$

This coincidence (22) once more confirms the statement in QDM that at first SDI effectively forms the diquark as a separate object and only after this action the further calculations of Λ structure may be carried out without spin-dependence forces. This fact was emotionally and impressively formulated by S. Fredriksson (one of the active elaborators of the QDM): "**Hello diquark, goodbye gluon!**" [13].

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МОДЕЛЬ БАРІОНІВ НА ОСНОВІ КВАРКІВ І ДІКВАРКІВ ТА ЕЛЕКТРОМАГНІТНА СТРУКТУРА ГІПЕРОНА Λ^0 (1115)

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У рамках кварк-дікваркової моделі (QDM) наведено результати розрахунків середньоквадратичного радіуса заряду (MSCR) гіперона Λ^0 (1115). Числові значення $< r_\Lambda^2 >_{QDM}$ суттєво залежать від маси m_{D_0} скалярного діварка $D_0 = (ud)$ $(J = 0)$, і тому MSCR Λ(QDM) може бути як додатним, так і $(-0{,}14\ fm^2 \leq < r_\Lambda^2 >_{QDM} \leq 0{,}20\ fm^2)$. Але, окрім Λ^0, скалярний діварк D_0 також одночасно входить до QD-структури протона та нейтрона. Отже, значення $< r_\Lambda^2 >_{QDM} = +0{,}06\ fm^2$ із m_{D_0} =0,434 ГеВ є кращим через необхідність правильного відтворення експериментальних значень радіуса заряду протона та MSCR нейтрона. Також коротко обговорюється магнітний момент Λ(QDM).