ELECTROMAGNETIC STRUCTURE OF $\Sigma^+, \Sigma^-, \Xi^0, \Xi^-$, AND Ω^- HYPERONS IN CONSTITUENT QUARK MODELS

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Electromagnetic structure of hyperons in the ground state with the strangeness $S = -1, -2, -3$ and with relatively long-living time ($\tau \sim 10^{-10}\ s$) is studied in two constituent quark models. The results of new calculations of mean-square charge radii, the masses and magnetic moments are presented. Analysis is made both for the pure $3q$-model and for the model that accounts of the spin-dependent interaction. The main conclusions and the general tendencies in hyperon series are formulated.

Earlier in the papers [1, 2] we made the detailed analysis of electromagnetic structure of Λ^0 (1115) hyperon in nonrelativistic constituent quark model (NRCQM) in two variants: usual (*uds*) model and the model with accounting the spin-depending interaction (SDI). These papers contain useful specific methods of calculations. The notations and definitions are also present in [1, 2]. Here we apply the analogous methods to investigate the hyperons mentioned in the papers' title.

Identity of the hyperon (H) quark structures: five hyperons $H = \Sigma^+, \Sigma^-, \Xi^0, \Xi^-, \Omega^-$ have the same:

- mass structure $H(q_1 q_2 q_3) = H(mmm_3)$,
- charge structure $H(q_1 q_2 q_3) = H(QQQ_3)$;
- spin structure $H(q_1 q_2 q_3) = H(J_{12} = 1, S_3 = 1/2)$, where $\vec{J}_{12} = \vec{S}_1 + \vec{S}_2$ is a total spin of the quark #1 and #2.

These properties simplify further calculations. All five hyperons belong to the family of (relatively) long-living hadrons with $\tau \sim 10^{-10}$ s. $\Sigma^0(uds)$-hyperon is not included to the list as it has very short lifetime $\tau \sim 10^{-20}$ s due to dominating radiation decay $\Sigma^0 \rightarrow \Lambda^0 + \gamma$.

The results of our calculations are summarized in Tables 1-3.

Table 1

Hyperon mean-square charge radii (MSCRi) in NRCQM and SDI contributions to the total MSCRi from renormalized contact Fermi-Breit potential

Hyperon	Radii in fm^2			
	NRCQM $\langle r^2 \rangle_{3q}$	SDI contributions $\langle \delta r^2 \rangle_{SDI}$	Total $\langle r^2 \rangle_t$	Experiment
Λ^0 (1115)	0.12	− 0.06	0.06	
Σ^+ (1189)	0.91	− 0.04	0.87	
Σ^- (1197)	0.68	− 0.08	0.60	0.61±0.12(stat.)±0.09(syst.)
Ξ^0 (1314)	0.22	− 0.13	0.09	
Ξ^- (1321)	0.63	− 0.096	0.53	
Ω^- (1672)	0.57	+ 0.07	0.64	

Table 2

Hyperon masses with spin-dependent interaction contribution Δ_{FB}

Hyperon	Masses in MeV				
	$\sum_{i=1}^{3} m_{qi}$	Δ_{FB}	m_{th} (theory)	m_{ex} (experiment)	$\left(\frac{m_{ex} - m_{th}}{m_{ex} + m_{th}}\right)$ %
Λ^0 (1115)	1170	− 56	1114	1115.683±0.06	0.08
Σ^+ (1189)	1170	− 38	1132	1189.37±0.07	2.5
Σ^- (1197)	1170	− 38	1132	1197.449±0.030	2.8
Ξ^0 (1314)	1350	− 39	1311	1314.86±0.20	0.15
Ξ^- (1321)	1350	− 39	1311	1321.71±0.07	0.4
Ω^- (1672)	1530	+ 33	1563	1672.45±0.29	3.4

Table 3

Magnetic moments (in μ_N units): octet of the lightest baryons + Ω^- hyperon

Baryon	Theory	Experiment	$\left\|\frac{m_{ex} - m_{th}}{m_{ex} + m_{th}}\right\|$ %
proton	2.793*	2.793*	*input (for determination of the lightest quark masses $m_u = m_d = 336$ MeV)
neutron	− 1.861	− 1.913	1.4
Λ^0	− 0.613*	− 0.613*	*input (for determination of the strange quark mass $m_s = 510$ MeV)
Σ^+	+ 2.686	+ 2.458±0.010	4.4
Σ^-	− 1.04	− 1.160±0.025	5.5
Ξ^0	− 1.438	− 1.250±0.014	7
Ξ^-	− 0.5073	− 0.6507±0.0025	12
Ω^-	− 1.84	− 2.02±0.05	4.7

The main conclusions and the general tendencies in the hyperon series in Table 1 may be formulated as follows.

1. MSCRi of all hyperons (H) are stable positive: $\langle r_H^2 \rangle > 0$.

Thus, for any hyperon the neutron phenomenon ($\langle r_n^2 \rangle \leq 0$) is not reproduced.

2. The full (total) value of $\langle r_{\Sigma^-}^2 \rangle_t$ appears to be very near (almost coincides) to the measured in 2021 experimental one.

It's especially interesting to mark that this coincidence is achieved only due to the account of the SDI contribution $\langle \delta r_{\Sigma^-}^2 \rangle_{SDI}$ to $\langle r_{\Sigma^-}^2 \rangle$. The "pure" $3q$-value $\langle r_{\Sigma^-}^2 \rangle_{3q}$ lies far enough from the experimental one.

3. Radii of all hyperons do not exceed the proton size: $r_H < r_p$.

The only exception is Σ^+; its radius is slightly larger than r_p:

$r_{\Sigma^+} \cong 0.93\, fm > r_p = 0.84\ or\ 0.87\, fm.$

4. MSCRi of neutral hyperons do not exceed the value of order $0.1\, fm^2$:

$\langle r^2 \rangle\, (\Lambda^0, \Sigma^0, \Xi^0) \lesssim 0.1\, fm^2.$

5. In NRCQM without SDI the MSCRi of the negatively charged hyperons Σ^-, Ξ^- and Ω^- have the tendency to decrease with increasing of the strange content in these hyperons:

$\langle r^2 \rangle_{3q} = (0.68;\ 0.63; 0.57)\, fm^2.$

6. The inclusion of SDI and meson exchanges contributions to MSCRi smooths out this tendency.

7. The contributions of SDI to the total MSCRi are not too small and therefore do not treat as perturbations. For SDI in the form of FBP it is caused by the relatively large both the value of renormalized coupling constant A and the light quarks mass factor $\left(1/m_i m_j\right)$ for the hyperons family.

For hard c- and b-quarks the values of $\langle\, \delta r^2 \rangle$ of charmed and bottom baryons decrease rapidly and may consider as small perturbations (as Fermi-Breit corrections to the Coulomb potential obtained for positronium in QED).

8. Table 2 clearly shows that the calculated and experimental hyperon masses coincide with accuracy to the few percents. It means that the numerous and complex calculations of kinetic (E_k) and potential (E_p) energies of quarks in the hyperon are not always necessary as we may expect almost the full mutual compensation of separate energies in the sum ($E_k + E_p$). In particular this is the case in Λ^0 hyperon.

Many years ago, Zeldovich and Sakharov [3] supposed that in the simplest version of quark model the masses of the s-wave mesons and baryons in ground states are the sum of constituent quark masses plus the term which describes the quark's spin-spin interaction. In this naïve model the E_k and E_p are totally ignored, but the authors hoped that the model opens the simple constructive possibilities to calculate the masses with enough accuracy (~20 MeV) in comparison with experimental values.

Now we can see that the authors were very close to the truth and Table 2 confirms it.

9. Comments to the Table 3. The estimates of hyperon's magnetic moments (MMs) in s-wave NRCQM are based on the simplest basic assumption:

MM $\mu(B)$ of any baryon B is formed as algebraic linear combination of the Dirac MMs of the proper constituent quarks:

$$\mu(B) = \sum_{i=1}^{3} C_i \mu_{q_i} = \sum_{i=1}^{3} C_i \frac{e_{q_i}}{2m_{q_i}},$$

where C_i are the numerical coefficients which have to be calculated separately for the each baryon.

From the analysis of the Table 3 we may conclude that:

- the deviations of the theoretical values from the experimental ones are of the variable quantity but don't exceed the value $|0.2\, \mu_N|$;
- theoretical and experimental values of MMs Σ, Ξ, Ω^- hyperons coincide with the accuracy of $\lesssim 10\%$;
- this result may be considered as more than satisfactory, especially taking into account the roughness of the basic assumptions.

In fact, the general situation is such that in any theory (even in the most advanced and refined ones) the calculated theoretical values of baryons MMs practically never coincide with the experimental ones (in this connection see, e.g., the discussion of Λ^0 MM in [1]). The definite discrepancy in MMs is always presented in any quark dynamics. But here we are convinced that NRCQM with the simple algebraic addition of Dirac MMs of the pointlike constituent quarks provides us with the reliable foundation for understanding the overwhelming part of MMs values. Therefore, in our opinion, the attempts for further improvement of the already obtained MMs are not so urgent or important.

In this connection, it's appropriate to remind the N. Isgur's and G. Karl's remark from the ref. [4]: "Our main conclusion therefore is that the fine details of baryon magnetic moments depend on too many unknowns to be settled at this time. Nevertheless, we find some consolation in the fact that main features of these moments have been predicted by the simplest quark models, and that observed deviation can be accommodated – though not reliably calculated – within the model".

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ЕЛЕКТРОМАГНІТНА СТРУКТУРА ГІПЕРОНІВ $\Sigma^+, \Sigma^-, \Xi^0, \Xi^-$ ТА Ω^- У МОДЕЛЯХ, ЩО БАЗУЮТЬСЯ НА СКЛАДОВИХ КВАРКАХ

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У двох моделях, що базуються на складових кварках, досліджено електромагнітну структуру гіперонів у основному стані зі странністю S=-1, -2, -3 та відносно тривалим часом життя ($\tau\sim10^{-10}$ с). Наведено результати нових розрахунків середньоквадратичних радіусов заряду, мас та магнітних моментів. Проведено аналіз як для чистої 3q-моделі, так і для моделі, що враховує спін-залежну взаємодію. Сформульовано основні висновки та загальні тенденції в рядах гіперонів.