DEPENDENCE OF THE AMPLITUDE OF THE WAKEFIELD ON THE SHAPE OF THE DRIVER-BUNCH

I.O. Afanacieva[1]*, D.O. Maslo*[1]*, V.I. Maslov*[2]
[1]*V.N. Karazin Kharkiv National University,*
Kharkiv, Ukraine;
[2]*Deutsches Elektronen-Synchrotron DESY,*
Hamburg, Germany
E-mail: zojnikua@gmail.com

The amplitude of the accelerating wakefield is important in overcoming the self-injection threshold. In this paper, the dependence of the amplitude of the accelerating wakefield on the shape of the electron bunch, exciting the wakefield, is investigated using numerical simulation and analytical methods. Five typical shapes of identically charge of bunches are examined: a uniform cylinder, Gaussian-like rising and falling semi-cosines, a rising triangle bunch, and a falling triangle bunch. It is shown that the more concentrated the electrons in the bunch, the larger the wakefield they excite.

PACS: 29.17.+w; 41.75.Lx

INTRODUCTION

The formation of the idea of using plasma fields to accelerate charged particles began back in the 1956. At that time, the development of this field was constrained by the limitations of the experimental base and the complexity of matching the driver parameters with the plasma parameters. But already in the 2000s, several groups managed to experimentally demonstrate the generation of beams in the self-injection regime. To date, two approaches to injecting electrons into the accelerating phase have emerged in the physics of wakefield acceleration: external injection and self-injection [1, 2].

1. EXTERNAL INJECTION

The method of external injection involves the use of two independently formed electron beams – a driver bunch and a witness bunch. It is important that both bunches are generated before injecting the plasma and have controlled parameters.

The driver beam can be either an intense laser pulse or a charged particle bunch with a large charge. It excites the plasma wave [1]. While moving through the plasma, the driver displaces the background electrons, as a result a cavity (bubble) filled with an ion background is formed behind it.

A second electron witness-bunch is directed following the driver [1, 3]. The main task is for it to get to the end of the formed cavity in the maximum accelerating field. Once there, the bunch is picked up by the electric field and accelerated.

Despite the fact that this method this method has advantages, its technical implementation is associated with challenges:

1. Time synchronization. This is a critical point: if witness-bunch arrives slightly earlier, it may fall into the decelerating phase [2, 4]. If it is late, the cavity will already close, and the witness-bunch will simply dissipate.
2. Positioning accuracy. The dimensions of the cavity are microscopic. It is necessary to perfectly inject the witness-bunch at the end of this region. An error of even a few microns will lead to the strong transverse fields ejecting the beam from the acceleration zone, destroying its structure.

2. SELF-INJECTION

An alternative approach is the self-injection regime, because here we do not need a second accelerator – the plasma itself becomes the source of the necessary particles.

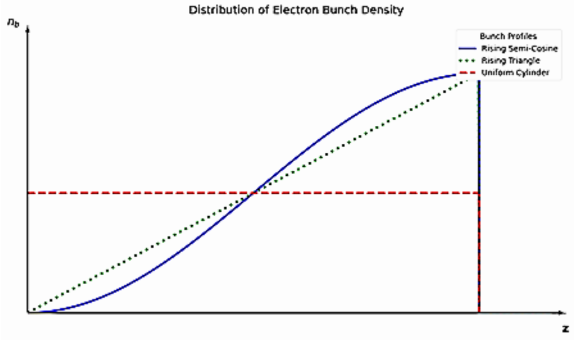
The essence of the process is that such a powerful wakefield is created that it begins to “break” (wave breaking) [1]. When the field exceeds a certain threshold, the plasma electrons on the walls of the cavity can no longer simply oscillate – they break away and are pulled inside cavity themselves. This is a self-consistent process: the system automatically injects charge into the accelerating phase.

However, despite the obvious advantage of this method, in practice there is a problem of controlling the parameters: beam quality and energy spread. In conventional self-injection, electrons can fly into the cavity chaotically and continuously. Some are trapped earlier and have time to gain high velocity, others – later and remain slow. As a result, at the output, there is a risk getting not a distinct beam, but a bunch with different energies, which is unacceptable for many applications.

Difficulty in reaching the threshold. Often, fields turn out to be insufficient to launch the self-injection process, or the process occurs unstably.

It is known that there is an effective way to facilitate self-injection by using a plasma density gradient [1, 5]. Since the wakefield wavelength depends on the plasma density (inversely proportional to the square root of the density), when using a decreasing gradient, the wavelength begins to increase. In this case, the front of the cavity moves at the speed of the driver, and the rear front begins to lag due to the “blowing up” of the cavity. This lag significantly facilitates the conditions for trapping plasma electrons inside the accelerating phase.

In our paper, we consider another way. To facilitate self-injection and reach the necessary threshold values, we use the dependence of the accelerating field amplitude on the shape of the driver bunch [6–8].



Distribution of electron bunch density

An important characteristic of the self-injection regime is its pronounced threshold character [1]. In the normal state, plasma electrons oscillate around the equilibrium position, forming a density wave. At the same time, they are confined by the Coulomb restoring force and do not leave their orbits. For self-injection to occur, the amplitude of the wakefield must exceed a certain critical value – the so-called wave breaking threshold. Laser-plasma accelerator experiments have demonstrated acceleration gradients > 100 GV/m [1]. Only when the field becomes supercritical, the trajectories of the electrons curve so much that they intersect. At this moment, the accelerating force overcomes the restoring force, and electrons break off into the cavity.

It is in this connection that the problem of the wave excitation efficiency arises. After all, reaching this threshold value simply by increasing the driver charge is not always possible or feasible.

It is known that the magnitude of the excited field depends not only on the total charge of the bunch, but also on its spatial configuration (shape). In this work, we investigate the dependence of the accelerating wakefield amplitude on the longitudinal profile of the driver [7–9]. Our goal is to find such a bunch shape that, at a fixed charge, will maximize the field, overcome the wave breaking threshold, and provide conditions for stable self-injection.

DETERMINATION OF THE MAXIMUM WAKEFIELD DEPENDING ON THE SHAPES OF THE DRIVER BUNCHES

High-efficiency acceleration of electron bunches by the plasma wakefield was studied experimentally, by numerical simulation and analytically [10–23].

To ensure stable self-injection, it is critically important to overcome the wave breaking threshold by generating a sufficiently high accelerating wakefield. Since the amplitude of the field significantly depends on the longitudinal profile of the driver, our goal is to determine the bunch shape that maximizes the accelerating field E_{acc} at a fixed total charge.

Let us consider 5 driver bunches with the same charge and length $L = \frac{\lambda_p}{2}$ (Figure and Table). We investigated the following driver profiles [6, 9]:

1. Uniform cylinder;
2. Rising half-cosine;
3. Rising triangle;
4. Falling half-cosine;
5. Falling triangle.

Case	Maximum positive value of the electric field on the axis E_z(decelerating field E_{dec})	Minimum value of the electric field (accelerating field E_{acc})	Transformation ratio
Uniform cylinder bunch	$8.0911 \cdot 10^{-4}$	$-1.6192 \cdot 10^{-3}$	$R = \frac{1.6192 \cdot 10^{-3}}{8.0911 \cdot 10^{-4}} = 2.001$
Rising half-cosine bunch	$1.2618 \cdot 10^{-3}$	$-2.0541 \cdot 10^{-3}$	$R = \frac{2.0541 \cdot 10^{-3}}{1.2618 \cdot 10^{-3}} = 1.628$
Rising triangle bunch	$1.0234 \cdot 10^{-3}$	$-1.9146 \cdot 10^{-3}$	$R = \frac{1.9146 \cdot 10^{-3}}{1.0234 \cdot 10^{-3}} = 1.871$
Falling half-cosine bunch	$1.2993 \cdot 10^{-3}$	$-2.0419 \cdot 10^{-3}$	$R = \frac{2.0419 \cdot 10^{-3}}{1.2193 \cdot 10^{-3}} = 1.572$
Falling triangle bunch	$1.1715 \cdot 10^{-3}$	$-1.9044 \cdot 10^{-3}$	$R = \frac{1.9044 \cdot 10^{-3}}{1.1715 \cdot 10^{-3}} = 1.626$

One can see from the table, the uniform cylinder profile provides the lowest value of the accelerating field, which may be insufficient to trigger self-injection [3, 24]. In contrast, profiled bunches demonstrate significantly higher field amplitudes under the same initial conditions.

For a uniform cylinder $j(\xi) = 1$ на $[0;\ \pi]$

Field inside $(0 \le \xi \le \pi)$

$$E_{in}(\xi) = \int_0^{\xi} 1 \cdot \cos(\xi - \xi')\, d\xi' = sin(\xi),$$

$$E_{in} = \sin\left(\frac{\pi}{2}\right) = 1.$$

Field outside$(\xi > \pi)$

$$E_{out}(\xi) = \int_0^{\pi} 1 \cdot \cos(\xi - \xi')\, d\xi',$$

$$E_{out}(\xi) = 2\sin(\xi).$$

Rising triangle $(n_b(\xi) = \xi)$

Field inside

$$E_{in}(\xi) = \int_0^{\xi} \xi' \cos(\xi - \xi') d\xi',$$

$$E_{in}(\xi) = 1 - cos(\xi).$$

Field outside

$$E_{out}(\xi) = \int_0^{\pi} \xi' \cos(\xi - \xi')\, d\xi',$$

$$E_{out}(\xi) = \pi \sin(\xi) - 2\cos(\xi).$$

Rising half-cosine $\boldsymbol{p(\xi) = 1 - cos\ (\xi)\ 0} \leq \xi \leq \pi$

Field inside

$$E_{in}(\xi) = \int_0^{\xi} (1 - \cos\xi')\, cos(\xi - \xi')\, d\xi',$$

$$E_{in}(\xi) = \frac{1}{2}\sin(\xi) - \frac{1}{2}\xi\cos(\xi).$$

Field outside

$$E_{out}(\xi) = \int_0^{\pi} (1 - \cos\xi') \cos(\xi - \xi')\, d\xi',$$

$$E_{out}(\xi) = 2sin\xi - \frac{\pi}{2}cos\xi.$$

Falling half-cosine $\boldsymbol{j(\xi) = 1 + cos\xi}$

Field inside

$$E_{in}(\xi) = \int_0^{\xi} (1 + \cos\xi')\ \cos(\xi - \xi') d\xi',$$

$$E_{in}(\xi) = 1.5\sin(\xi) + 0.5\xi\cos(\xi).$$

Field outside

$$E_{out}(\xi) = \int_0^{\pi} (1 + \cos\xi') \cos(\xi - \xi')\, d\xi',$$

$$E_{out}(\xi) = 2sin\xi + \frac{\pi}{2}\cos(\xi).$$

Falling triangle $\boldsymbol{j}(\xi) = 2 - \frac{2}{\pi}\xi'$

Field inside

$$E_{in}(\xi) = \int_0^{\xi} \left(2 - \frac{2}{\pi}\xi'\right) cos(\xi - \xi')\, d\xi',$$

$$E_{in}(\xi) = 2\sin(\xi) - \frac{2}{\pi}(1 - \cos(\xi)).$$

Field outside

$$E_{out}(\xi) = \int_0^{\pi} \left(2 - \frac{2}{\pi}\xi'\right) cos(\xi - \xi')\, d\xi',$$

$$E_{out}(\xi) = 2\sin(\xi) + \frac{4}{\pi}\cos(\xi).$$

Optimization of the driver shape: ideal bunch

Despite the fact that the profiles considered above allow us to increase the amplitude of the accelerating field, their transformation ratio is quite strongly limited by the value$R \leq 2$. At the same time, the requirement for ensuring stable and effective acceleration is the need to achieve the maximum accelerating wakefield, but at the same time minimize the decelerating field.

Back in the 80s, a solution was found to achieve a high transformation ratio, which is the use of a rather specific "door-step" type [6, 7]. The logic of the shape is as follows:

1. Precursor – a uniform cylinder, which is equal to exactly a quarter of the plasma wave $L_1 = \frac{\pi}{2} \approx 1.57$. The main goal is to properly phase the plasma wave.

2. The second section is exactly the so-called "ramp". In this zone, our charge increases proportionally. This linear growth compensates for the harmonic nature of the plasma wave and creates an absolutely flat decelerating field inside the bunch.

According to calculations, the theoretical decelerating field is $E_{dec}^{theor} = 0.10$ and the accelerating one reaches $E_{acc}^{theror} = 0.3397$ [6]. Hence the theoretical transformation ratio is $R_{theor} = 3.397$. Obviously, this field overcomes the limit of 2.

The numerical simulation data also confirm this: $E_{dec}^{exp} = 1.0891 \cdot 10^{-3}$ $E_{acc}^{exp} = 3.4131 \cdot 10^{-3}$ which gives the transformation ratio in numerical simulation $R_{exp} = 3.134$.

The error between the numerical simulation and the theoretical calculation is only 8.4%, which indicates the obvious correctness of the model [3, 24].

CONCLUSIONS

A comparison of the obtained values shows that the lowest efficiency of field excitation is demonstrated by a bunch in the shape of a uniform cylinder. The maximum accelerating field in this case is $|E_{acc}| = 1.62$. Although this case provides the highest transformation ratio $R = 2$, the field amplitude itself may be insufficient to reach the wave breaking threshold necessary to trigger the self-injection process. In contrast, the use of profiled bunches allows significantly increasing the amplitude of the excited wave. In particular, drivers with a half-cosine profile demonstrate an increase in the maximum accelerating field to values of $|E_{acc}|\ = 2.05$.

Since for classical profiles the transformation ratio is fundamentally limited by the boundary $R \leq 2$, the optimal solution is the application of the "ideal bunch" considered above [6–8]. As the simulation results have shown, due to a special spatial configuration, this driver forms a uniform decelerating field and allows confidently overcoming the mentioned limit, reaching a transformation ratio of > 3.

Thus, the transition from standard drivers to complex-profiled bunches with a precursor is the most promising path both for reliably exceeding the wave breaking threshold and initiating stable self-injection, and for significantly increasing the overall efficiency of the plasma accelerator.

REFERENCES

1. E. Esarey, C.B. Schroeder, W.P. Leemans. Physics of laser-driven plasma-based accelerators // *Rev. Mod. Phys.* 2009, v. 81, №3, p. 1229-1285.

2. W.P. Leemans, B. Nagler, A.J. Gonsalves, C. Toth, K. Nakamura, C.G.R. Geddes, E. Esarey. GeV electron beams from a laser-wakefield accelerator // *Nat. Phys.* 2006, v. 2, №10, p. 696-699.

3. M. Litos et al. High-efficiency acceleration of an electron beam in a plasma wakefield accelerator // *Nat.* 2014, v. 515, p. 92.

4. W.P. Leemans, A.J. Gonsalves, H.-S. Mao, et al. Multi-GeV electron beams from capillary-discharge-guided subpetawatt laser pulses in the self-trapping regime // *Phys. Rev. Lett.* 2014, v. 11, p. 245002.

5. V.I. Maslov, O.M. Svystun, I.N. Onishchenko, A.M. Yegorov. Joint wakefield acceleration by laser pulse and by self-injected electron bunches // *Probl. At.*

Sci. Technol. Series: "Plasma Physics". 2016, N 6, p. 144.

6. V.I. Maslov, I.N. Onischenko, Transformation ratio at wakefield excitation in dielectric resonator by sequence of rectangular electron bunches with linear growth of charge // *Probl. At. Sci. Technol.* 2014, N 3 (91), p. 95.
7. V.I. Maslov, I.N. Onischenko. Transformation ratio at wakefield excitation in dielectric resonator accelerator by shaped sequence of electron bunches with linear growth of current // *Probl. At. Sci. Technol*. 2013, N 4 (86), p. 69.
8. V.I. Maslov, I.N. Onischenko, I.P. Yarovaya. Transformation ratio at excitation of nonlinear wake field in plasma by shaped sequence of electron bunches with linear growth of charge // *Probl. At. Sci. Technol*. 2012, N 4 (80), p. 126.
9. V.I. Maslov, I.N. Onischenko, I.P. Yarovaya. Wakefield excitation in plasma by ramped sequence of ramped electron bunches // *Probl. At. Sci. Technol*. 2012, N 6(82), p. 161.
10. T. Tajima, J.M. Dawson. Laser Electron Accelerator // *Phys. Rev. Lett*. 1979, v. 43, p. 267.
11. T. Tajima. Laser Acceleration in Novel Media *Eur. Phys. J. Spec. Top*. 2014, v. 223, p. 1037.
12. V.I. Maslov et al. Plateau formation on accelerating wakefield for electron-witness-bunch and on decelerating wakefield for driver bunches // *Problems At. Sci. Technol*. 2021, N 6, p. 52-56.
13. D.O. Shendryk et al. Simulation of the identical plateaus formation on plasma wakefield for long driver and witness // *Problems At. Sci. Technol*. 2023, N 6, p. 65.
14. I.V. Demydenko, V.I. Maslov. Identical decelerating wakefield for driver and identical accelerating wakefield for witness for their periodic sequence // *Problems At. Sci. Technol.* 2023, N 3, p. 108-112.
15. D.S. Bondar, V.I. Maslov, I.N. Onischenko. A method for maintaining the acceleration rate and increasing the energy of selfinjected bunch due to the use of inhomogeneous plasma // *Probl. At. Sci. Technol.* 2023, N 4, p. 67-71.
16. M. Tzoufras et al. Beam loading in the nonlinear regime of plasmabased acceleration // *Phys. Rev. Lett*. Sep. 2008, v. 101, N 14, Art. no. 145002.
17. V.I. Maslov et al. Control of characteristics of self-injected and accelerated electron bunch in plasma by laser pulse shaping on radius, intensity and shape // *Problems At. Sci. Technol.* 2019, N 6, p. 39-42.
18. V. Maslov et al. Numerical simulation of Plateau formation by an electron bunch on the distribution of an accelerating wakefield in a plasma // *Probl. At. Sci. Technol*. 2020, N 6, p. 47-49.
19. S. Romeo et al. Beam loading assisted matching scheme for high quality plasma acceleration in linear regime // *Phys. Rev. Accel. Beams.* 2020, v. 23, p. 071301.
20. V.I. Maslov et al. Plateau formation on accelerating wakefield for electron-witness-bunch and on decelerating wakefield for driver-bunches in a plasma // *Probl. At. Sci. Technol*. 2021, N 6, p. 52-56.
21. A. Martinez de la Ossa et al. Intrinsic stabilization of the drive beam in plasma-wakefield accelerators // *Phys. Rev. Lett.* 2018, v. 121, p. 064803.
22. G. Loisch et al. Towards experimental investigation of hosing instability mitigation at the PITZ facility // *J. Phys., Conf. Ser*. 1596 (2020) 012003.
23. E. Adli et al. Transverse tolerances in the plasma-wakefield acceleration blowout regime *// Proc. 16th Int. Part. Accel. Conf.* 2025, p. 1434-1437.
24. C.A. Lindstrøm, J.M. Garland, S. Schröder, et al. Energy-spread preservation and high efficiency in a plasma-wakefield accelerator // *Phys. Rev. Lett*. 2021, v. 126, p. 014801.

ЗАЛЕЖНІСТЬ АМПЛІТУДИ КІЛЬВАТЕРНОГО ПОЛЯ ВІД ФОРМИ ЗБУДЖУЮЧОГО ЗГУСТКА

І.О. Афанасьева, Д.О. Масло, В.І. Маслов

Амплітуда прискорюючого кільватерного поля є важливою у проблемі подолання порога самоінжекції. У цій доповіді досліджено числовим моделюванням і аналітично залежність амплітуди прискорюючого кільватерного поля від форми згустка електронів, що збуджує кільватерне поле. Досліджено п'ять типових форм згустка однакового заряду: однорідний циліндр, гауссоподібні наростаючий і спадаючий напівкосинуси, згусток – наростаючий трикутник і згусток – спадаючий трикутник. Показано, чим більш сконцентровані електрони в згустку, тим більше кільватерне поле вони збуджують.