

KINEMATIC CHARACTERISTICS OF RESONANT SCATTERING OF AN ELECTRON BY A MUON IN AN ELECTROMAGNETIC WAVE FIELD

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The conditions of resonant scattering of an electron by a muon in the field of an intense electromagnetic wave when multiphoton processes and intensity effects become significant are investigated. The resonance corresponds to the falling of an intermediate photon on the mass surface.

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INTRODUCTION

In paper [1] the resonant scattering of an electron by a muon was studied in the case of weak wave intensity: $\eta \ll 1$, where η is wave field intensity parameter [2, 3]. In this case, partial process in which one photon is absorbed from the wave in the first subprocess, and one is emitted into the wave in the second subprocess is significant. Other partial processes can be neglected since the interaction with one additional wave photon introduces a small multiplier $\sim \eta^2$.

In order to find the probability of the resonant approximation in the process of electron scattering by a muon in the field of an intense linearly polarized wave, an analysis of the conditions of the resonant course of the process was carried out. In this case, it is necessary to take into account other partial processes that may demonstrate resonant behavior. Furthermore, the analysis is complicated by the fact that the conservation laws must take into account additions to the momentum due to the influence of the electromagnetic wave. Thus, instead of momentum, the conservation laws include quasimomenta of particles. In the case of linear polarization, the quasi-momentum is determined by the formula:

$$\tilde{p} = p + (m^2\eta^2/2)k, \quad \tilde{p}^2 = \tilde{m}^2, \quad (1)$$

where $\tilde{m} = m(1 + \eta^2/2)$ is effective mass of a particle in a wave field; m is mass of particle. Let m be the mass of the electron; m_μ be the mass of the muon.

Note that in this paper we consider the case when an intermediate resonant photon is emitted by an electron and then absorbed by a muon. The obtained results are easily generalized to the chronologically opposite case, when the intermediate resonant photon is emitted by a muon.

ANALYSIS OF RESONANCE CONDITIONS

The amplitude of the electron scattering process by a muon in the field of an electromagnetic wave can be represented as the sum of partial terms, each of which corresponds to the conservation laws:

$$\tilde{p}_i + l_1 k = \tilde{p}_f + \kappa, \quad (2)$$

$$\tilde{\Pi}_i + \kappa = \tilde{\Pi}_f + l_2 k, \quad (3)$$

where $\tilde{p}_{i,f}$ are quasimomenta of the initial and final electrons; $\tilde{\Pi}_{i,f}$ are quasimomenta of the initial and final muons; $k = (\omega, \mathbf{k})$ is a wave 4-vector; κ is 4-momentum of an intermediate photon (in general: $\kappa^2 \neq 0$); the integer l_1 is the number of wave photons absorbed from

the wave in the first subprocess, l_2 is the number of wave photons emitted from the wave in the second subprocess. Formula (2) corresponds to the conservation law in the 1st vertex, formula (3) corresponds to the conservation law in the 2nd vertex.

Eliminating κ from the system (2), (3), we obtain the conservation law:

$$\tilde{p}_i + \tilde{\Pi}_i + lk = \tilde{p}_f + \tilde{\Pi}_f, \quad (4)$$

where $l = l_1 - l_2$ are the total number of photons absorbed from the wave in the process.

The resonance corresponds to the condition of the intermediate photon being on the mass surface:

$$\kappa^2 = 0. \quad (5)$$

Partial processes that can occur in a resonant manner:

$$l \geq 0, l_1 \geq 1, l_1 \geq l, l_2 = l_1 - l. \quad (6)$$

Note that there are 2 particles in the final state, which require 6 parameters to describe. The presence of four δ -functions in expression (4) allows us to determine four of them. We will take the angular characteristics of the scattered electron as independent: $\mathbf{n}_f = \mathbf{n}_f(\theta_f, \psi_f)$. The resonance condition fixes one additional variable.

To find the resonance condition, we will follow the frequency of the intermediate photon, i.e. the equation for it is linear, while the equation for the energy of the final electrons is biquadratic. That is, its characteristics cannot be observed, so this is an analogue of parametric coordinate specification.

Let us square the left and right sides of the equations of the system (2), (3) and take into account the resonance condition (5) and solve the obtained equalities with respect to the resonant frequency of the intermediate photon. As a result, we find:

$$\kappa_{0,res} = \frac{l_1(kp_i)}{(|\tilde{p}_i + l_1 k|n_\kappa)}, \quad \kappa_{0,res} = \frac{l_2(k\Pi_i)}{(|\tilde{\Pi}_i - l_2 k|n_\kappa)}. \quad (7)$$

Equating the resulting expressions, we find:

$$\frac{l_1(kp_i)}{(|\tilde{p}_i + l_1 k|n_\kappa)} = \frac{l_2(k\Pi_i)}{(|\tilde{\Pi}_i - l_2 k|n_\kappa)} \Rightarrow (hn_\kappa) = 0, \quad (8)$$

$$\cos \angle(\mathbf{h}, \mathbf{n}_\kappa) = \frac{h_0}{|\mathbf{h}|}, \quad (9)$$

$$h = (h_0, \mathbf{h}) = \frac{\tilde{p}_i + l_1 k}{l_1(kp_i)} - \frac{\tilde{\Pi}_i - l_2 k}{l_2(k\Pi_i)}. \quad (10)$$

Therefore, the resonant condition for the process of electron scattering by a muon in a wave field corresponds to the directions of the intermediate photon lying on the surface of the cone, the axis of which coincides with the vector \mathbf{h} , and the semi-vertical angle:

$$\angle(\mathbf{h}, \mathbf{n}_\kappa) = \alpha, \quad \alpha = \arcsin\left(\frac{h_0}{|\mathbf{h}|}\right), \quad (11)$$

It also follows from (9) that the resonance condition is met by the parameters of the initial particles that satisfy the condition:

$$h^2 \leq 0. \quad (12)$$

That is, the 4-vector h is a space-like. Inequality (12) is equivalent to the condition:

$$u_i \leq \rho_i, \quad (13)$$

where invariant parameters u_i, ρ_i are obtained

$$u_i = \frac{(x-1)^2}{4x}, \rho_i = \tau + \frac{1}{2} \left(\frac{u_i}{\mu} + \mu U_l \right), \quad (14)$$

$$x = \frac{u_{l_1}}{\mu U_{l_2}}, \tau = \frac{(\tilde{\Pi}_i \tilde{p}_i)}{\tilde{m} \tilde{m}_\mu}, \mu = \frac{\tilde{m}_\mu}{\tilde{m}}. \quad (15)$$

The parameter u_l , introduced in [3], and its generalization to the muon case have the form

$$u_l = l u_1, u_1 = \frac{2(k p_i)}{\tilde{m}^2}, \quad (16)$$

$$U_l = l U_1, U_1 = \frac{2(k \Pi_i)}{\tilde{m}_\mu^2}. \quad (17)$$

The parameter u_i , as a function of x in the region of positive values, has an extremum at the value $x = 1$: $u_{i,min} = 0$. The parameter u_i takes its maximum value at the limit values x (15):

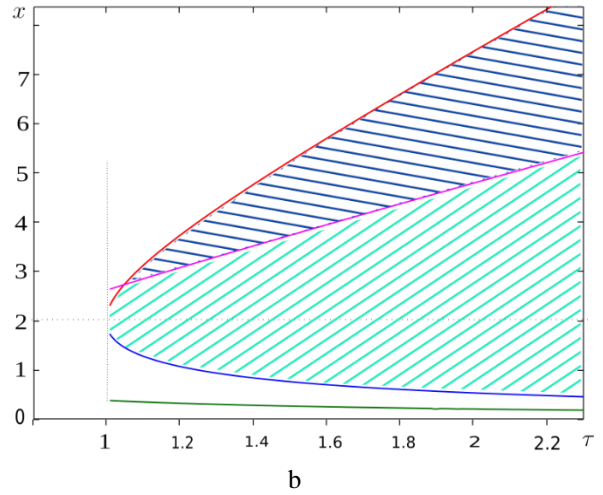
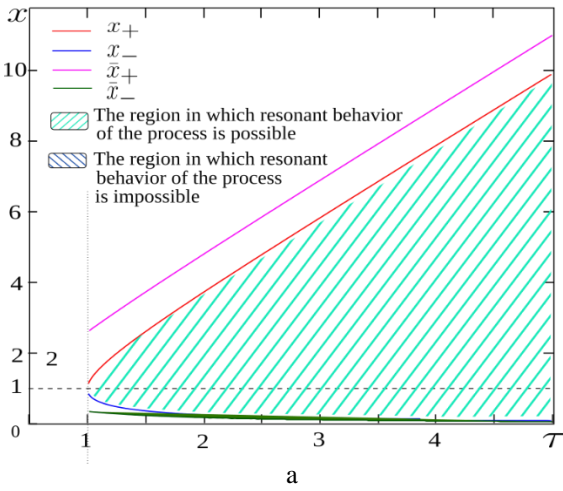
$$x_- \leq x \leq x_+, x_\pm = \frac{l_1}{l_2} (\tau \pm \sqrt{\tau^2 - 1}). \quad (18)$$

On the other hand, inequality (1) implies a restriction on the parameter x , which limits the region where the resonant behavior of the process is possible:

$$\bar{x}_- \leq x \leq \bar{x}_+; \bar{x}_\pm = t \pm \sqrt{t^2 - 1}, \quad (19)$$

$$t = \tau + \frac{1}{2} \left(\frac{u_i}{\mu} + \mu U_l \right). \quad (20)$$

Due to inequality $t > \tau$ for partial processes $l_1 = l_2$ ($l = 0$) range of permissible parameter values x (18) is located within the area (19). That is, for all permissible values of the parameter x , a resonant behavior of the process is possible (Figure, a).



The variation range of parameter x for $\frac{u_l}{\mu} + \mu U_l = 1$:
a - $l = 0$ ($l_1 = l_2 = 1$); b - $l = 1$ ($l_1 = 2, l_2 = 1$)

At the same time, for partial processes $l \geq 1$ ($l_2 < l_1$) we have an upper bound for the parameter x (Figure, b):

$$1 \leq \tau \leq \tau_*, x_- \leq x \leq x_+; \quad (21)$$

$$\tau > \tau_*, x_- \leq x \leq \bar{x}_+. \quad (22)$$

For parameter values $\tau > \tau_*$ exists a region where a resonant regime of the process is impossible (see Figure, b):

$$\bar{x}_+ \leq x \leq x_+. \quad (23)$$

From the geometry of the emission of the intermediate photon, we can find the corresponding resonant frequency:

$$\kappa_{0,res} = \frac{l_1(k p_i)}{Q_{i,0} - |\mathbf{Q}_i| \cos \angle(\mathbf{Q}_i, \mathbf{n}_\kappa)}, \quad (24)$$

where $Q_i = \tilde{p}_i + l_1 k$, and $\cos \angle(\mathbf{Q}_i, \mathbf{n}_\kappa)$ can be written as follows:

$$\cos \angle(\mathbf{Q}_i, \mathbf{n}_\kappa) = \cos \alpha \cos \theta_Q + \sin \alpha \sin \theta_Q \cos(\varphi - \varphi_Q), \quad (25)$$

where $\theta_Q = \angle(\mathbf{h}, \mathbf{Q}_i)$; φ_Q is the azimuthal angle of the vector \mathbf{Q}_i in a spherical coordinate system where the polar angle is measured from the vector \mathbf{h} .

We obtain the value of angle θ_Q using the formula:

$$\cos \theta_Q = \frac{(\mathbf{h} \mathbf{Q}_i)}{|\mathbf{h}| |\mathbf{Q}_i|} = \frac{1 - \frac{(\hbar Q_i)}{h_0 Q_{i,0}}}{\sqrt{\left(1 - \frac{\hbar^2}{h_0^2}\right) \left(1 - \frac{Q_i^2}{Q_{i,0}^2}\right)}}. \quad (26)$$

Assuming the condition $\omega \ll m$ holds for laser systems and taking the initial particles to be ultrarelativistic: $E_i \gg m, \Pi_i \gg m_\mu$, we obtain:

$$\alpha \approx \frac{\sqrt{-\hbar^2}}{h_0} \approx \frac{\tilde{m}}{\tilde{E}_i} \delta_\alpha, \quad (27)$$

$$\delta_\alpha = \frac{4 u_{l_1} U_{l_2}}{(\mu U_{l_2} - u_{l_1}) \left(U_{l_2} - \frac{u_{l_1}}{\mu^2} \frac{\tilde{\Pi}_{i,0}}{\tilde{E}_i} \right)} \times \sqrt{u_i (\rho_i - u_{l_1})}, \quad (28)$$

$$\theta_Q \approx \sqrt{\left(\frac{\hbar}{h_0} - \frac{Q_i}{Q_{i,0}} \right)^2} \approx \frac{\tilde{m}}{\tilde{E}_i} \delta_Q, \quad (29)$$

$$\delta_Q \approx \sqrt{\delta_\alpha^2 + \frac{(\hbar Q)}{u_{l_1} \mu^2 U_{l_2}} - 1 - u_{l_1}}, \quad (30)$$

$$(\hbar Q) = \frac{2(1+u_{l_1})}{u_{l_1}} - \frac{2\mu\tau}{l_2 U_1} + \frac{u_1}{U_1} - \mu^2 \frac{l_1}{l_2}. \quad (31)$$

For the resonant frequency of the intermediate photon (in units of the quasienergy of the initial electron) under these conditions we obtain

$$\frac{\kappa_{0,res}(\psi)}{\bar{E}_i} \approx \frac{u_{l_1}}{1+u_{l_1}+\delta_\alpha^2+\delta_Q^2-2\delta_\alpha\delta_Q\cos(\psi-\psi_Q)}. \quad (32)$$

From the parameters of the intermediate photon, we can derive the parameters of the scattered electron. Thus, the quasienergy of the scattered electron (in units of the initial electron's quasi-energy) is equal to

$$\frac{\bar{E}_f(\psi)}{\bar{E}_i} \approx 1 - \frac{\kappa_{0,res}(\psi)}{\bar{E}_i}. \quad (33)$$

The angle at which the electron is emitted after scattering, in a system where the polar angle is measured from the vector \mathbf{h} , is equal to

$$\cos \tilde{\theta}_f = \frac{(\tilde{\mathbf{p}}_f \mathbf{h})}{|\tilde{\mathbf{p}}_f||\mathbf{h}|} = \frac{|\mathbf{Q}_i| \cos \theta_Q - \kappa_{0,res} \cos \alpha}{\bar{E}_f \sqrt{1-\tilde{m}^2/\bar{E}_f^2}}, \quad (34)$$

$$\tilde{\theta}_f = \frac{\tilde{m}}{\bar{E}_i} \tilde{\delta}_f, \quad (35)$$

$$\tilde{\delta}_f^2 = \frac{\bar{E}_i}{\bar{E}_f} \left(\delta_Q^2 - \frac{\kappa_{0,res}}{\bar{E}_i} \delta_\alpha^2 - \frac{\bar{E}_i^2}{\bar{E}_f^2} \right). \quad (36)$$

The polar angle of the scattered electron can be found from the equation:

$$\tan \tilde{\theta}_f = \frac{\delta_Q \cos \psi_Q - (\kappa_{0,res}/\bar{E}_i) \delta_\alpha \cos \psi}{\delta_Q \sin \psi_Q - (\kappa_{0,res}/\bar{E}_i) \delta_\alpha \sin \psi}. \quad (37)$$

CONCLUSIONS

The investigation of the conditions for the resonant of electron-muon scattering in the field of an intense electromagnetic wave has shown

1. The resonant behavior demonstrates partial processes for: $l \geq 0$, $l_1 \geq 1$, $l_1 \geq l$, $l_2 = l_1 - l \geq 1$, where l_1 corresponds to the number of wave photons

absorbed from the wave in the first subprocess, l_2 corresponds to the number of wave photons emitted from the wave in the second subprocess, l are the total number of photons absorbed from the wave in the process.

2. The directions of the intermediate photon in the resonance region form a cone, the axis of which is determined by the vector \mathbf{h} (10) and the semi-vertical angle α (11). For ultrarelativistic initial particle energies, the radiation occurs within a narrow cone: $\alpha \sim \tilde{m}/\bar{E}_i \ll 1$.

3. The frequency of the intermediate photon is determined by expression (32) and varies within the interval:

$$\frac{u_{l_1}}{1+u_{l_1}+(\delta_\alpha+\delta_Q)^2} \leq \frac{\kappa_{0,res}}{\bar{E}_i} \leq \frac{u_{l_1}}{1+u_{l_1}+(\delta_\alpha-\delta_Q)^2}.$$

4. Based on the known characteristics of the intermediate photon that define the resonance region, the characteristics of other final particles can also be found. In particular, the energy and scattering directions of the electron are determined by expressions (33)–(37).

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КІНЕМАТИЧНІ ОСОБЛИВОСТІ РЕЗОНАНСНОГО РОЗСИВАННЯ ЕЛЕКТРОНА НА МІОНИ У ПОЛІ ЕЛЕКТРОМАГНІТНОЇ ХВИЛІ

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Досліджено умови резонансного розсіювання електрона на міюні у полі інтенсивної електромагнітної хвилі, коли стають суттєвими багатофотонні процеси та ефекти інтенсивності. Резонанс відповідає виходу проміжного фотона на масову поверхню.