CALCULATION OF THE DISPERSION PERFORMANCES OF SLOW WAVE STRUCTURES WITH ARTIFICIAL ANISOTROPICAL DIELECTRICS

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The cylindrical waveguide periodically loaded with dielectric disks [1] represents the waveguide of slow waves which can find application in the technique of amplification and generation of electromagnetic waves, as well as in accelerating technique. The waveguides with a dielectric load are effective in a wide range of phase velocities v_{ph} ($\beta = v_{ph}/c \sim 0{,}1\text{-}1{,}0$) and have extra properties favourably distinguishing them from structures with an all metal load. And though they have the some limitations caused by the known behaviour of dielectrics in high-frequency fields of the high strength, the structures with such advantages can attract attention of the developers of new high-frequency technique.

In development of such an electrophysical equipment one of basic problems is to satisfy the requirements of synchronism between the velocity of charged particle beam and the phase velocity of electromagnetic wave propagating in the structure. The aim of the present paper is to study the dispersion characteristics of the waveguide loaded with dielectric disks with central holes for passing the particle beam.

Let us consider the metal waveguide of circular section of radius R, periodically loaded with dielectric disks of a thickness b, distance between disks is equal to a, L = a+b, period of disk arrangement. The permittivity of disk material is identical and equal to ε, between disks ε = 1. The radius of the pass channel is designated as r_0. The losses of electromagnetic energy in a metal wall of the waveguide and in a dielectric volume in sectional operation are not taken into account. In the waveguide the symmetric wave of E-type propagates (below the time factor $\exp(i\omega t)$ is omitted). The solution of this problem will be carried out in a general form where the structure period is comparable with the wavelength in the waveguide ($L \sim \lambda_g$).

In a case, when the load consists of solid dielectric disks ($r_0 = 0$), the dispersion properties of the waveguide are featured by the well- known equation [2]

$$\cos\psi = \cos(p_1 a)\cos(p_2 b) - \frac{1}{2}\left(\frac{\varepsilon p_1}{p_2} + \frac{p_2}{\varepsilon p_1}\right)\sin(p_1 a)\sin(p_2 b), \qquad (1)$$

where $p_1^2 = k^2 - k_1^2$, $p_2^2 = \varepsilon k^2 - k_1^2$ is the propagation constants in separate layers of the waveguide load, $k = \omega / c$ - wave number, $k_1 = \sigma_0 / R$ - transversal wave number, σ_0 - first root of cylindrical functions of the first order, $\psi = \omega L / v_{ph} = kL/\beta$ - shift of the phase in the structure period. From this equation for given values of geometrical and electrotechnical parameters it is possible to determine a phase velocity of a wave in the waveguide.

The situation varies radically, when in the waveguide it is necessary to have the channel for passing particles interacting with a field of a slow wave. The presence of holes in disks leads to decrease of a total dielectric load of the waveguide and to redistribution of a field existing in the waveguide at $r_0 = 0$. To solve the dispersion equation at $r_0 \neq 0$ we shall take advantage of a procedure formulated in [3]. The dispersion equation obtained has, as in the case of the waveguide loaded with metal diaphragms, a form of an infinite determinant but the form of the field expansion in separate regions and character of convergence of the solution obtained are completely various.

The field in the pass channel is represented as an infinite decomposition on space harmonics

$$E_я^I = \sum_{n=-\infty}^{\infty} a_n J_n(\chi_n r) e^{-i\beta_n z},$$

$$H_\varphi^I = \sum_{n=-\infty}^{\infty} a_n \frac{ik}{\chi_n} J_1(\chi_n r) e^{-i\beta_n z}, \qquad (2)$$

where $\chi_n^2 = k^2 - \beta_n^2$, and the values of β_n are interrelated to a stationary value of distribution of the first harmonic $\beta_0 = \omega / v_0$ by a Floquet's relation $\beta_n = \beta_0 + 2\pi n / L$. A field in the annular region is represented as the expansion in terms of space harmonics of the cylindrical waveguide periodically loaded with solid dielectric disks

$$E_z^{II} = \sum_{m=-\infty}^{\infty} c_m Z_0(\Gamma_m r) \frac{W_m(z)}{\varepsilon(z)},$$

$$H_\varphi^{II} = \sum_{m=-\infty}^{\infty} c_m \frac{ik}{\Gamma_m} Z_1(\Gamma_m r) W_m(z). \qquad (3)$$

Here A^e_{mn} and A^m_{mn} are coefficients of the expansion

$$A^e_{mn} = \frac{1}{L}\int_0^L e^{i\beta_n z} \frac{W_m(z)}{\varepsilon(z)} dz,$$

$$A^m_{mn} = \frac{1}{L}\int_0^L e^{i\beta_n z} W_m(z) dz. \qquad (4)$$

$W_m(z)$ - periodic function with a period L, describing the dependence of a field in the waveguide on the longitudinal coordinate. For the wave, propagating in positive direction of then axis z, the function $W_m(z)$ with unit starting conditions can be written as

$$W_m(z) = u_1(z) - \frac{u_1(L) - e^{-i\psi}}{u_2(L)} u_2(z). \qquad (5)$$

where the functions $u_1(z)$ and $u_2(z)$ represent the fundamental solutions of the equation

$$\varepsilon(z)\frac{d}{dz}\frac{1}{\varepsilon(z)}\frac{dW_m}{dz} + [k^2\varepsilon(z) - \Gamma^2]W_m = 0. \qquad (6)$$

The radial part in the expansion (3) is determined with a combination of cylindrical functions of the zero and first orders

$Z_0(\Gamma_m r) = N_0(\Gamma_m R)J_0(\Gamma_m r) - J_0(\Gamma_m R)N_0(\Gamma_m r)$,

$Z_1(\Gamma_m r) = N_0(\Gamma_m R)J_1(\Gamma_m r) - J_0(\Gamma_m R)N_1(\Gamma_m r)$.

Equating the corresponding harmonics at $r = r_0$ we come to the following set of equations concerning required coefficients a_n and c_m:

$$a_n J_0(\chi_n r_0) = \sum_{m=-\infty}^{\infty} c_m Z_0(\Gamma_m r_0) A^e_{mn} ,$$

$$\frac{a_n}{\chi_n r_0} J_1(\chi_n r_0) = \sum_{m=-\infty}^{\infty} c_m \frac{1}{\Gamma_m r_0} Z_1(\Gamma_m r_0) A^m_{mn} . \qquad (7)$$

The condition for solving the obtained homogeneous set of algebraic equations is the equality to zero of its determinant

$$\left| \frac{1}{\chi_n r_0} \frac{J_1(\chi_n r_0)}{J_0(\chi_n r_0)} - \frac{1}{\Gamma_m r_0} \frac{Z_1(\Gamma_m r_0)}{Z_0(\Gamma_m r_0)} \frac{A^m_{mn}}{A^e_{mn}} \right| = 0 . \quad (8)$$

which is the required dispersion equation of the waveguide loaded with dielectric disks with central holes. Similarly to the case of the waveguide loaded with metal diaphragms, it looks like an infinite determinant. Note, that the convergence of the determinant obtained will be determined by a behavior of functions, on which the expansion is carried out [4]. In the used approach the decomposition of fields is carried out on eigenfunctions of the inhomogeneous cylindrical waveguide, therefore convergence of the dispersion equation will be the better, the smaller radius of a central hole in disks is.

To demonstrate peculiarities of calculations for dispersion properties of such waveguides we consider, as an example, the calculation of the dielectric disk thickness as a function of the radius value of a central hole in the waveguide with the given value of the phase velocity (β = const). The waveguide is specified by the following parameters: $\varepsilon = 90$, $R = 4{,}2$ cm, working wavelength $\lambda = 10$ cm. In the waveguide the travelling wave propagates with a phase velocity $\beta = 0{,}4$ and phase shift $\psi = \pi/2$ in a structure period.

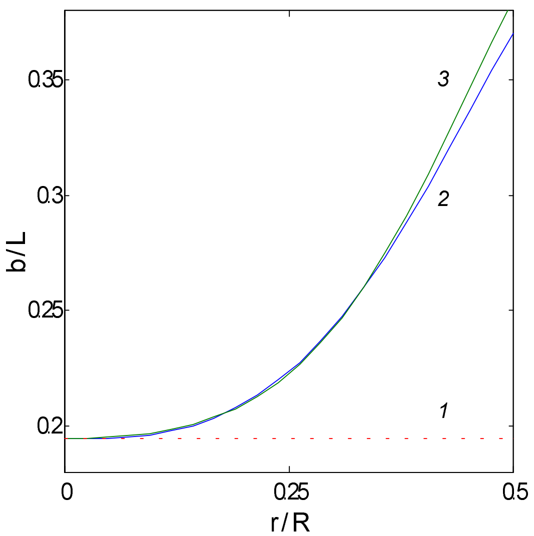

Fig. 1.

In Fig. 1 the change of a fill factor of a structure period with the dielectric b/L is shown depending on the radius of a central hole r_0/R. The curve 1 corresponds to the change b/L at loading the waveguide with solid dielectric disks, 2 - calculation for the zero term of the equation (8) and 3 - calculation for the complete dispersion equation (8). From this consideration it follows, that at $r_0/R < 0{,}1$ dispersion properties of structure do not differ from properties of waveguide loaded by solid disks, but at $r_0/R > 0{,}15$ it is necessary to take into account the changed structure of the field, i.e. to take into account the presence of the spectrum of space harmonics.

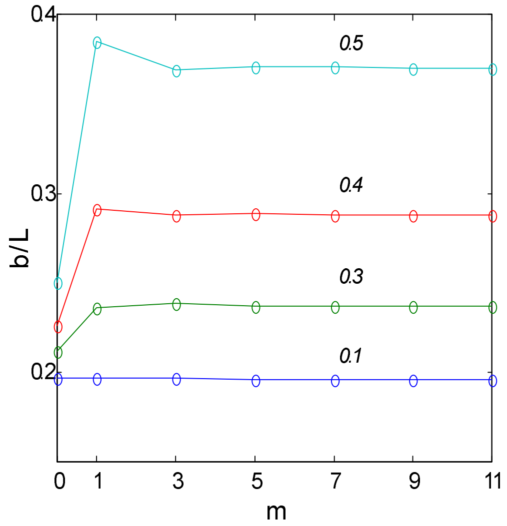

Fig. 2.

In Fig. 2 the character of convergence of the solution of the dispersion equation (8) is shown depending on the order of a computed determinant. In this figure the reduced radius of the central hole is taken as a parameter. From theconsideration it follows, that with calculations being sufficiently accurate for practice (~ 1 micron) one may restrict itself to the solution of the determinant with m = 5.

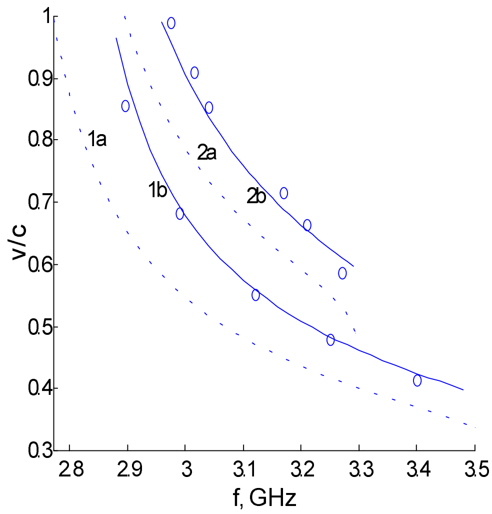

Fig. 3.

Fig. 3 represents the results of numerical calculations of dispersion curve for the waveguide (solid) in comparison with these experimentally obtained (points) taken from the paper [5] For this case $r_0/R = 1/3,6$: $\varepsilon = 80$. The curve 1b corresponds to the waveguide loaded with dielectric disks of a thickness b = 3 mm with a period of arrangement L = 1,2 cm; a curve 2b - b = 4 mm and L = 2,2 cm. The dotted lines 1a and 2a show dispersion characteristics for the corresponding waveguides loaded by solid dielectric disks. From the above dependence it follows that the data obtained correspond each other within the limits of an experimental error. That proves the chosen approach to the solution of a problem in distribution of electromagnetic waves in the circular metal waveguide loaded with dielectric disks having central holes.

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