ELECTRODYNAMICS OF HELICAL SLOWING STRUCTURE WITH HIGH-CURRENT ELECTRON BEAM

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In most theoretical and experimental research, devoted to studying the interactions of beams with hybrid structures, the beam density is assumed small, therefore it does not change a structure of eigenwaves of electrodynamic system the beam interacts with. It is interesting to consider another limiting case, when the beam density is already so large, that natural oscillations of beam particles (ω_b) are more than frequency of electromagnetic waves exited by beam, and when the presence of beam significantly changes the slowing structure electrodynamics.

STATEMENT OF A PROBLEM

In the present report we shall investigate theoretically and numerically the dispersion characteristics and find increments of instability of nonequilibrium systems with beams of large density that consist of annular electron beam moving along axes of helical slowing structure, which is immersed in strong external magnetic field directed parallel to the system axis.

Let us consider a helical waveguide with radius R_h, with helix period λ_h and pitch angle of helix ψ immersed in the strong external magnetic field $H_0 \| Z$. Along axis (axis Z) the annular electron beam with current I_{bo} is moved being localized near the mean radius $R_b < R_h$ in the rather narrow region with thickness $\Delta << R_h$, so the electrons density is constant in transverse direction and can be described by the function:

$$n_b(r) = \frac{I_{bo}}{eV_o 2\pi R_b \Delta}\theta(R_b - r + \frac{\Delta}{2})\theta(r - R_b + \frac{\Delta}{2}),$$

where $\theta(x)$ is the unit Heaviside function, V_o is the equilibrium beam velocity along the system axis, I_{bo} is the beam current, $2\pi R_b \Delta = S_b$ is its cross section. The electromagnetic fields in the system is described usually by Maxwell's equations. Let assume, that the perturbed quantities varies as $f(r)\exp i(k_{\|}z - \omega t)$.

THE BOUNDARY CONDITIONS

When solving the electrodynamics problem about long waves propagation ($\lambda >> \lambda_h$) we consider a helix as infinitesimal thickness and perfectly conducting anisotropy cylinder with standard boundary conditions for fields (see, for example, [1-3]):

$$E_h^{(1)} = 0; \quad E_h^{(2)} = 0;$$
$$E_r^{(1)} = E_r^{(2)}; \quad H_h^{(1)} = H_h^{(2)}. \quad (1)$$

The index (2) and (1) are for fields inside and outside of helix, accordingly, h and τ are direction along and perpendicular wires in a plane tangent to helix. The boundary conditions at thin-wall annular beam consist of the condition for continuity of tangential components of fields and presence of jump in the azimuthal magnetic field due to the beam current:

$$E_{\varphi,z}^{(1)} = E_{\varphi,z}^{(2)}; \quad H_z^{(1)} = H_z^{(2)}; \quad (2)$$

$$H_\varphi^{(3)} - H_\varphi^{(2)} = iR_b \frac{\omega}{c}\frac{\omega_b^2}{(\omega - k_{\|}V_o)^2}E_z(R_b), \quad (3)$$

where: $\omega_b^2 = 4\pi e^2 n_{bo}/m$ is the plasma frequency of beam, n_{bo}, V_o is the equilibrium density and beam velocity, respectively, m_o is the mass of electron, c is the velocity of light. Since we assume, that the beam is rather thin, and in the system the oscillations with a wavelength $\lambda >> \Delta$ are propagated, it is possible do not take into account a beam stratification in transverse direction. The value of the field acting on electrons can be taken in a point equal to the mean beam radius.

THE DISPERSION EQUATION

Producing matching of the fields according to boundary conditions (1-3) we obtain the dispersion equation of the thin annular beam-helix system [4]:

$$D_h D_b = M_o(\kappa R_h) F_h F_b, \quad (4)$$

where:

$$D_h = k_o^2 - \kappa^2 F_h, \; D_b = 1 - M_o(\kappa R_b)F_b,$$

$$F_b = \frac{\omega_b^2}{(\omega - k_{\|}v_o)^2}(\kappa R_b)^2 I_o^2(\kappa R_b),$$

$$F_h = tg^2\Psi \frac{I_o(\kappa R_h) K_o(\kappa R_h)}{I_1(\kappa R_h) K_1(\kappa R_h)},$$

and $\kappa = \sqrt{k_{\|}^2 - k_o^2}$, $M_o(x) = \frac{K_o(x)}{I_o(x)}$, $k_o = \frac{\omega}{c}$.

In this case the equation $k_o^2 - \kappa^2 F_h = 0$ is the dispersion equation of the «cold» vacuum helix (without electron beam), and the equation $1 - M_o(\kappa R_b)F_b = 0$ is the dispersion equation of thin annular electron beam in vacuum. When right-hand side of Eq. (4) is small the dispersion equation is decomposed, naturally, to two independent equations for eigenwaves of helix and beam:

$$D_h = 0, \; D_b = 0. \quad (5)$$

From Eq. (5) follow relations for phase velocities of helical and beam modes:

$$\beta_{1,2} = \pm\beta_h, \; \beta_{3,4} = \frac{\beta_o}{1 + \Omega_b^2} \pm \sqrt{1 + \frac{1 - \beta_o^2}{\Omega_b^2}}, \quad (6)$$

where: $\Omega_b^2 = \omega_b^2 G$ is the reduced beam density, $G = (R_b^2/c^2) I_o(\kappa R_b) K_o(\kappa R_b)$ is the beam depression coefficient characterizing its space-charge,

$\beta_h = (F_h / (1+F_h))^{1/2}$ is the phase velocity of the helix without beam, $\beta_o = V_o / c$.

From Eq.(6) follows, that the beam density influence becomes significant at $\Omega_b > \beta_o$. This relation allows to find the value of beam density, in dependence on the geometry and its velocity, since which the beam influence becomes dominant. In dimensionless variables $\beta = \omega / k_{||} c$, β_o, β_h the equation (4) will be:

$$(\beta^2 - \beta_h{}^2)[(\beta - \beta_o)^2 - \omega_b^2 G(1-\beta^2)] = \mu \omega_b^2 \quad (7)$$

Here $\mu = G(1-\beta^2)^2 \beta_h{}^2 M_o(\kappa R_h) / M_o(\kappa R_b)$ is the coefficient of beam-wave coupling. If the beam interacts with a forward wave of helix (propagated along the beam), then the coupling coefficient in a right-hand side of (7) is positive, and is negative in the case of interaction with a backward wave.

In general case the analysis of dispersion equation (7) can be carried out numerically, First of all, we will be interested in cases of slow waves propagation ($\beta^2 << 1$, so $k_{||} >> k_o$; $k_{||} \approx \kappa$). Thus, the equation (7) becomes much easier – from transcendental it turns into algebraic with respect to β:

$$(\beta^2 - \beta_h{}^2)[(\beta - \beta_o)^2 - \Omega_b{}^2] = \alpha \beta_h{}^2 \Omega_b{}^2 \quad (8)$$

where: $\alpha = M_o(k_{||} R_h) / M_o(k_{||} R_b)$ (magnitude $\alpha < 1$).

For further analysis it will be convenient to use the frame connected with beam $\beta = \beta_o + \delta$, $\beta_h = \beta_o + \Delta$ and equation (8) can be written as:

$$\frac{\beta_h{}^2 \alpha}{(2\beta_o + \Delta + \delta)(\delta - \Delta)} = \left(\frac{\delta^2}{\Omega_b{}^2} - 1 \right) \quad (9)$$

Right-hand side of this equation corresponding to fast and slow beam waves is the usual parabola. The left-hand side has poles of first order in points $\delta = \Delta$ and $\delta = -\Delta - 2\beta_o$ are corresponding to forward and backward wave of helix, and maximum in the point $\delta = -\beta_o$ which value is equal $-\alpha$.

In order to understand in what conditions the instability disappears, it is useful to plot the left-hand and right-hand sides of equation (9) on the graph (Fig. 1). It is easy to see that the solution of this equation always contains two real roots in the region of positive and negative values δ:

$$\min(-(\beta_s + \beta_o), -\Omega_b) > \delta$$

$$\max(\ (\beta_s - \beta_o),\ \Omega_b) < \delta$$

Whereas $abs(\alpha) < 1$, at values of the beam density larger then some critical value ($n_b > n_b{}^*$) $\Omega_b{}^2 > \beta_h{}^2 / (1-\alpha)$ and all roots of the equation (8) become real.

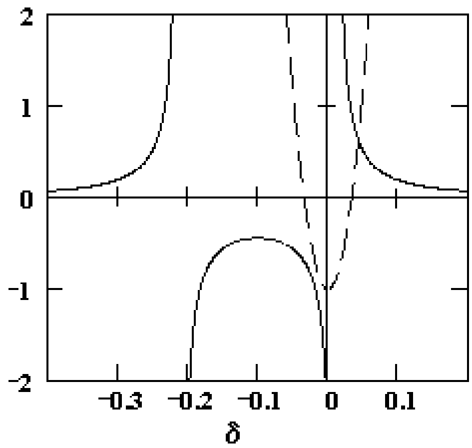

Fig. 1. Plot of the right-hand side (dashed line) and left-hand side (solid line) of the beam-loaded dispersion equation in the frame of beam.

Under small beam densities $\beta^2 >> \Omega_b{}^2$ we search a solution of the equation (8) near to intersection of beam and helical modes $\beta = \beta_0 + \delta$, $\beta_h = \beta_0$ and thus obtain:

$$\delta\ (\delta^2 - \Omega_b{}^2) = (1/2)\alpha \beta_h \Omega_b{}^2 \quad (10)$$

When $\delta^2 >> \Omega_b{}^2$ we find the ordinary cubic increment of beam instability:

$$\mathrm{Im}\,\delta = i(\sqrt{3}/2^{4/3})(\alpha \beta_h)^{1/3} \Omega_b{}^{2/3}. \quad (11)$$

If the beam density grows ($\Omega_b{}^2 > \delta^2$) so, that the beam influence on waves propagation in the system becomes significant, the Cherenkov's instability disappears: $\delta = -1/2\alpha\beta_h$.

From the view point of physics one can explain it in such a way. When the beam density increases the splitting of its dispersion curve to fast and slow beam modes becomes so large, that the dispersion curves of helix and beam do not intersect.

Now the instability is possible in the case, when the beam velocity V_o is more than wave phase velocity in the system $\beta_o > \beta$. Really, supposing that conditions $\beta = \beta_h + \delta$, $\beta_h = \beta_o - \Omega_b$ are fulfilled, we can note, that the second condition is the condition of anomalous Doppler radiation. Then from (8) we find a quadratic increment (typical for instability on anomalous Doppler): $\mathrm{Im}\,\delta = i/2\,(\alpha \beta_h \Omega_b)^{1/2}$.

RESULTS OF NUMERICAL ANALYSIS

For numerical analysis of this equation the following parameters were selected: $\beta_h = 0.1$, $k_{||} R_b = 3$, and ratio $R_h / R_b = 1.1$. The numerical solution's results of the equation (7) one can see in fig.2-3 as a dependence of normalized phase velocity ($V_{ph} / V_h = (\mathrm{Re}\,\beta) / \beta_h$) and increment ($\gamma = (\mathrm{Im}\,\beta) / \beta_h$) on the beam density ν_b ($\nu_b \equiv \omega_b{}^2 R_b{}^2 / c^2 \beta_h{}^2$) for various values of detuning between the beam velocity and wave phase velocity in the system ξ ($\xi = \beta_o / \beta_h$) for $\xi = 1.0$, $\xi = 1.1$, $\xi = 1.2$.

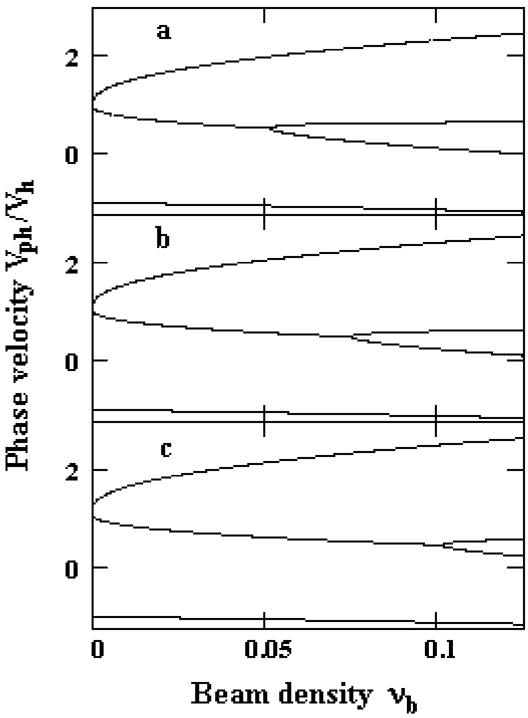

Fig. 2. Normalized phase velocity versus beam density for the varies value of detuning ξ: (a) – ξ =1.0, (b) – ξ =1.1, (c) – ξ =1.2.

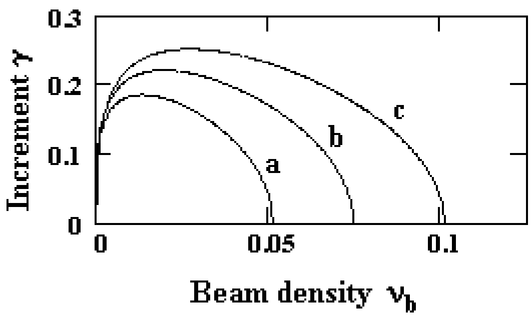

Fig. 3. Normalized increment versus beam density for various values of detuning ξ: (a) – ξ =1.0, (b) – ξ =1.1, (c) – ξ =1.2.

From these plots one can see the following most important regularities for influence of the parameter ξ and the beam density on V_{ph} and γ change:

For the given detuning ξ there is some beam density bounded range space at which the beam instability develops and wave excitation by beam takes place.

With growth of the beam density at some beam density values there is a maximum increment $\gamma = \gamma_{\max}$. At the further growth of beam density the value γ decreases and at $\nu_b{}^2 = \nu_b^{*2}$ the instability disappears, i.e. at these values $\nu_b{}^2 > \nu_b^{*2}$ there are no more degeneration in the system – phase velocities of two waves which were equal earlier, become various now.

With growth of the detuning parameter ξ the maximum values of increment and the values of beam density at which this maximum can be reached increase.

Significant changing of phase velocities of waves propagated along the beam takes the place. Two waves which propagate along the beam are essentially slowing (in comparison with a velocity of wave in the helical waveguide without a beam), third is the fast.

The numerical analysis also shows that with growth of the velocity detuning ξ the beam influence on the phase velocity of backward wave is decreased. But even at $\xi \sim 1$ the beam influence on the backward wave phase velocity is relatively small and is distinct only at rather large values of beam density. This result is in a qualitative agreement with analytical investigations carried out above.

CONCLUSION

Thus, we have carried out analytical investigation and numerical analysis for dispersion chracteristics and have found increments of instability of nonequilibrium system – annular electron beam in helical slowing structure for a beam of large density, when the frequency of beam natural oscillations is more than frequency of oscillations excited by him. Values of beam density in dependence on geometry and of beam velocity at which the beam influence on dispersion is dominant, were determined analytically and numerically. It is shown, that the density growth leads not only to significant changes in dispersion properties of the system, but also to modification of the mechanism for generation of oscillations in the system - from Cherenkov's instability to instability on anomalous Doppler effect; the further beam density growth leads to the instability failure.

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